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Strategic Manipulations and Collusions in Knaster Procedure

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Abstract The Knaster's procedure is one of the simplest and most powerful mechanisms for allocating indivisible objects among agents requiring them, but its sealed bid feature may induce some agents in altering their valuations. In this paper we study the consequences of false declarations on the agents' payoffs. A misrepresentation of a single agent could produce a gain or a loss. So, we analyze a possible behavior of a subset of infinitely risk-averse agents and propose how to obtain a safe gain via a joint misreporting of their valuations, regardless of the declarations of the other agents.

Keywords Knaster's procedure, misrepresentation, collusion
JEL classification C70

1. Introduction

In the literature, the problem of the allocation of a set of indivisible items with monetary compensations was tackled with a procedural approach by many authors. We just mention the pivotal works of Knaster (1946) and Steinhaus (1948) and refer to Brams and Taylor (1996 and 1999) for a survey on more recent papers. For other topics related to division problems we address to Young (1994) and Moulin (2003).

The interest for a procedural approach descends from the possibility of a "*practical implementation of a fair division outcome, in particular when parties in real life prefer to establish fairness by themselves*" (see Haake, Raith and Su, 2002).

We deal with the Knaster's procedure, in which the values of the objects for each agent are assumed to be additive; this implies that for each agent the value of an obtained object is independent from who has obtained the other objects. One of the most important features of this procedure is that it is based on the valuations of the agents of the objects to be assigned; these valuations are presented via a sealed bid mechanism and are used to determine the objects received and the amount of the monetary compensation of each agent. We are motivated to devote our attention to the Knaster's procedure because it is simple to implement, easy to understand and, in case the agents truthfully report their valuations, satisfies two relevant theoretical properties:

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efficiency (there exists no other distribution that yields every agent a higher payoff) and proportionality (each of the *n* agents thinks to receive at least one *n*-th of the total value). For more details see Brams and Taylor (1996 and 1999).

As it was pointed out by Brams and Taylor (1996) "*a potential drawback of Knaster's procedure is that players can profitably misrepresent their preferences if they know the monetary values that the other player(s) attribute to the items*." On the other hand, false declarations can be risky as an agent does not know the secret declarations of the others.

In this paper we start with a deeper analysis of the misrepresentations of the valuations, computing the actual advantage (or disadvantage) of such manipulations, supposing that each agent knows only his valuation and does not make use of any statistical information on the valuations of the other agents. Next, we go further, showing that a proper subset (coalition) of at least two completely risk-averse agents may profit from an information exchange on their true valuations, and a consequent agreement on altering their declarations; in this way they may get a safe gain w.r.t. the situation of truthful declarations.

In the existing literature, several authors studied the problem of manipulation in allocation mechanisms that associate an allocation to each set of agents' declarations. In this context the notion of strategy-proofness (non-manipulability) means that truthful report is a dominant strategy (Svensson 1999 and 2009). According to Holmström (1979) there is no Pareto efficient and strategy-proof procedure, consequently the Knaster's procedure is manipulable, due to its efficiency.

Here, we consider a different concept of manipulation considering that an infinitely risk-averse agent wants to incur no risk, i.e. he wants always to gain by misrepresenting his valuation, independently from the declarations of the other agents. In this context the Knaster's procedure is non-manipulable.

Various authors considered situations in which agents may form coalitions jointly misreporting their valuations; they obtained other concepts of manipulation and corresponding concepts of coalition-strategy-proofness. For example, in Moulin (1993) a mechanism is coalition-strategy-proof when "*if a joint misreport by a coalition strictly benefits one member of the coalition, it must strictly hurt at least one (other) member*". According to this definition the Knaster's procedure is manipulable. In the following we consider another concept of coalition-manipulability, the collusion: A coalition can manipulate with no risk a procedure if its members can always gain by truthful revelation of their valuations, misrepresentation of the declarations and reallocation of the payoffs, independently from the declarations of the other agents. We show that a coalition of at least two infinitely risk-averse agents, but obviously not all, can manipulate the Knaster's procedure. Other concepts of manipulability and strategy-proofness can be found in Papai (2000) where the reallocation-proofness via swapping the assigned items is considered and in Serizawa (2006) that studies pairwise strategy-proofness. When monetary compensations are allowed, we may refer to Schummer (2000) for bribe-proofness and Papai (2003) for dominant strategy incentive compatibility when envyfreeness is a relevant aim.

The paper is organized as follows: the next section recalls the scheme of the Knas-

ter's procedure; Section 3 analyses the advantages and the risks for an infinitely (i.e. completely) risk-averse agent misrepresenting his true valuations; the characteristics of a collusive behavior are presented in Section 4; Section 5 concludes.

2. The Knaster's Procedure

In this section, we briefly recall the procedure proposed by Knaster (1946). We suppose that m indivisible objects $b_1, ..., b_m$, $M = \{1, ..., m\}$, have to be assigned to the agents of the set $N = \{1, ..., n\}$. The final result is the assignment of each object to just one agent with monetary compensations. Preliminarily, each agent $i \in N$ declares to a mediator (in a sealed bid) his valuations $v_{i1}, ..., v_{im}$ of the items; let $E_i = \frac{1}{n}\sum_{k\in M} v_{ik}$, i.e. E_i is the initial proportional share according to the valuations of agent i. Then the procedure goes as follows:

Step 1 each object b_k, $k \in M$ is assigned to an agent $j(k)$ that gives the maximal valuation, $j(k) \in \arg\max\ v_{ik}$, $i \in N$ (if more agents give the same maximal valuation of an object it is assigned randomly);

Step 2 for each agent $i \in N$, let $G_i = \sum_{k:j(k)=i} v_{j(k),k}$; $V_i = E_i + \frac{s}{n}$, where $s = \sum_{j\in N}(G_j - E_j)$;

Step 3 for each agent $i \in N$, if the monetary amount $V_i - G_i$ is positive the player i receives it in addition to his objects;
otherwise player i pays $G_i - V_i$.

Citing Brams and Taylor (1996), the surplus s is non-negative; the division is proportional, as for each $i \in N$, $V_i \geq E_i$; the sum of the compensations is zero, as $\sum_{j\in N}(G_j - V_j) = 0$, so the procedure does not require or produce money.

According to a private communication of Fink to Brams in 1994, mentioned in Brams and Taylor (1996), the Knaster's procedure can be rewritten considering object by object. In fact, the surplus s may be rewritten as

$$s = \sum_{k\in M}\left(v_{j(k),k} - \sum_{i\in N}\frac{1}{n}v_{ik}\right) = \sum_{k\in M} s_k,$$

where $s_k = v_{j(k),k} - \sum_{i\in N}\frac{1}{n}v_{ik} \geq 0$ for each $k \in M$.

So, it is easy to check that we obtain the same allocation with the Knaster's procedure and with the following procedure applied to each single item b_k, $k \in M$:

Step 1 the object b_k, $k \in M$ is assigned to an agent $j(k)$ that gives the maximal valuation which pays the monetary amount $v_{j(k),k}$ (if more agents give the same maximal valuation of the object it is assigned randomly);

Step 2 each agent $i \in N$ receives the amount $\frac{1}{n}v_{ik}$;

Step 3 the remaining amount s_k is equally shared among all the agents.

In the next section we study the possible misrepresentations of the valuations of the agents. According to what we said above, we can suppose to have to assign just one object, and study the consequences of manipulations for this object; eventually, we sum up the results for each item.

3. Misrepresentation

Suppose that the true valuations of a set of agents $N = \{1, ..., n\}$ for the unique object are denoted by v_i, $i \in N$. For sake of simplicity let $v_1 \geq v_2 \geq ... \geq v_n$ and in case of several agents giving the same maximal valuation, v_1, let 1 be the agent that gets the object, after the random assignment. Consequently, at the end of the procedure, agent 1 gets the object, pays $\frac{n-1}{n} v_1$ and receives $\frac{1}{n}\left(v_1 - \frac{1}{n}\sum_{j\in N} v_j\right)$, whereas each agent $i \in N \setminus \{1\}$ receives $\frac{1}{n} v_i + \frac{1}{n}\left(v_1 - \frac{1}{n}\sum_{j\in N} v_j\right)$.

Now, we suppose that one agent decides to make a different declaration for the object, with the aim of increasing his payoff, while the other agents do not modify their declarations. To make simpler this analysis we consider separately the two situations in which agent 1 or one of the other agents manipulates his declaration.

Case A. *Agent* 1 *decreases valuation; no change in allocation*

Agent 1 declares $v_1 - \varepsilon$ with $\varepsilon < v_1 - v_2$, so he still has the highest declaration. He gets the object, pays $\frac{n-1}{n}(v_1 - \varepsilon)$ and receives $\frac{1}{n}\left[(v_1 - \varepsilon) - \frac{1}{n}\sum_{j\in N} v_j + \frac{1}{n}\varepsilon\right]$, so the variation of his payoff is $\frac{(n-1)^2}{n^2}\varepsilon$; so, he has a gain only if $\varepsilon > 0$. Each agent $i \in N \setminus \{1\}$ receives $\frac{1}{n} v_i + \frac{1}{n}\left[(v_1 - \varepsilon) - \frac{1}{n}\sum_{j\in N} v_j + \frac{1}{n}\varepsilon\right]$ and his payoff reduces of $\frac{n-1}{n^2}\varepsilon$.

Case B. *Agent k increases valuation; no change in allocation*

Agent $k \in N \setminus \{1\}$ declares $v_k + \varepsilon$ with $\varepsilon < v_1 - v_k$, so agent 1 still has the highest declaration. The final amount of agent k is $\frac{1}{n} v_k + \frac{1}{n}\varepsilon + \frac{1}{n} v_1 - \frac{1}{n^2}\sum_{j\in N} v_j - \frac{1}{n^2}\varepsilon$; as the variation of the payoff is $\frac{n-1}{n^2}\varepsilon$, he has a gain only if $\varepsilon > 0$. Agent 1 gets the object, pays $\frac{n-1}{n} v_1$, receives $\frac{1}{n}\left(v_1 - \frac{1}{n}\sum_{j\in N} v_j - \frac{1}{n}\varepsilon\right)$ and his payoff reduces of $\frac{1}{n^2}\varepsilon$; each agent $i \in N \setminus \{1, k\}$ receives $\frac{1}{n} v_i + \frac{1}{n}\left(v_1 - \frac{1}{n}\sum_{j\in N} v_j - \frac{1}{n}\varepsilon\right)$ and his payoff reduces of $\frac{1}{n^2}\varepsilon$.

It is straightforward to check that in cases A and B the gain of the agent that modifies his valuations is equal to the sum of the losses of the other agents (note that the total allocated value is v_1). Moreover, the variations of the payoffs depend only on the variation of the declaration, ε, and on the number of agents, n, and is independent from the declarations of the other agents. Then, we investigate what happens when the variations are such that agent 1 no longer gets the object.

Case C. *Agent* 1 *decreases valuation; object allocated to agent* 2

Agent 1 declares $v_1 - \varepsilon$ with $\varepsilon > v_1 - v_2$ and the object goes to agent 2 that declares v_2; he receives $\frac{1}{n}(v_1 - \varepsilon) + \frac{1}{n}\left(v_2 - \frac{1}{n}\sum_{j\in N} v_j + \frac{1}{n}\varepsilon\right)$, suffering a loss of $\frac{1}{n}(v_1 - v_2) + \frac{n-1}{n^2}\varepsilon$. Agent 2 obtains the object, pays $\frac{n-1}{n} v_2$ and receives $\frac{1}{n}\left(v_2 - \frac{1}{n}\sum_{j\in N} v_j + \frac{1}{n}\varepsilon\right)$; the variation of his payoff is $\frac{1}{n^2}\varepsilon - \frac{1}{n}(v_1 - v_2)$; each agent $i \in N \setminus \{1, 2\}$ receives $\frac{1}{n} v_i + \frac{1}{n}\left(v_2 - \frac{1}{n}\sum_{j\in N} v_j + \frac{1}{n}\varepsilon\right)$ and his payoff varies of $\frac{1}{n^2}\varepsilon - \frac{1}{n}(v_1 - v_2)$.

Case D. *Agent k increases valuation; object allocated to agent k*

Agent $k \in N \setminus \{1\}$ declares $v_k + \varepsilon$ with $\varepsilon > v_1 - v_k$ and gets the object; he pays $\frac{n-1}{n}(v_k + \varepsilon)$ and receives $\frac{1}{n}\left[(v_k + \varepsilon) - \frac{1}{n}\sum_{j \in N} v_j - \frac{1}{n}\varepsilon\right]$; so, his payoff reduces of $\frac{1}{n}(v_1 - v_k) + \frac{(n-1)^2}{n^2}\varepsilon$. Agent 1 loses the object and obtains $\frac{1}{n}v_1 + \frac{1}{n}\left[(v_k + \varepsilon) - \frac{1}{n}\sum_{j \in N} v_j - \frac{1}{n}\varepsilon\right]$; the variation of his payoff is $\frac{n-1}{n^2}\varepsilon - \frac{1}{n}(v_1 - v_k)$; each agent $i \in N \setminus \{1,k\}$ receives $\frac{1}{n}v_i + \frac{1}{n}\left[(v_k + \varepsilon) - \frac{1}{n}\sum_{j \in N} v_j - \frac{1}{n}\varepsilon\right]$ and his payoff varies of $\frac{n-1}{n^2}\varepsilon - \frac{1}{n}(v_1 - v_k)$.

In case C, the difference between the new and the old payoff for the agents $i \in N \setminus \{1\}$ is positive if $\varepsilon > n(v_1 - v_2)$. In case D, the difference between the new and the old payoff for the agents $i \in N \setminus \{k\}$ is positive if $\varepsilon > \frac{n}{n-1}(v_1 - v_k)$. In both cases the false declaration results in a socially inefficient situation. In fact, if the agent $k \in N \setminus \{1\}$ gets the object, the sum of the payoffs of the agents is v_k and the assumption that $v_1, \ldots, v_n$ are the true valuations of the agents plays an important role.

We conclude the section with a simple example in order to better explain the previous results.

Example 1. Suppose that the true valuations of the four agents I, II, III and IV the objects are as in the following table:

I	II	III	IV
240	192	80	64

The object goes to agent I and the final payoffs are 84, 72, 44 and 40, respectively.

Now, we consider the following cases:

- A Agent I decreases his true declaration of 16 units
- B Agent II increases his true declaration of 16 units
- C Agent I decreases his true declaration of 208 units (losing the object)
- D Agent II increases his true declaration of 80 units (getting the object)
- C' Agent I decreases his true declaration of 64 units (losing the object)
- D' Agent II increases his true declaration of 56 units (getting the object)

The payoffs are in the following table:

	I	II	III	IV
True valuations	84	72	44	40
A	93	69	41	37
B	83	75	43	39
C	33	73	45	41
D	87	15	47	43
C'	60	64	36	32
D'	82.5	28.5	42.5	38.5

Note that in case C' the decreasing of 64 units of agent I is less than $n(v_1 - v_2) = 192$ and in case D' the increasing of 56 units of agent II is less than $\frac{n}{n-1}(v_1 - v_2) = 64$; consequently, the inefficient allocation of the object to agent II, causes a loss to all the agents.

Example 1 shows that the misrepresentation of the valuations may be risky for an agent if he does not know in advance the declarations of the other agents.

4. Collusion

In this section we study how a suitable information sharing and consequent agreement among at least two agents avoids the risk of a loss.

Definition 1. *A collusion of a coalition of completely risk-averse agents consists of:*
1) truthful revelation among them of their valuations;
2) same declaration of the highest true valuation;
3) binding agreement on the gain sharing.

In the following of the section, we show that if a coalition of completely risk-averse agents behaves according to the above definition it may obtain the highest safe gain, independently from the declarations of the other agents.

To make the reasoning simple, we start analyzing a 2-agent collusion. Suppose that agents i and j agree on altering their declarations and let v_i and v_j be their valuations, with $v_i > v_j$. As v_j cannot be the highest valuation (w.r.t. all the agents), it is not an advantage to decrease it. On the other hand, v_i could be the highest valuation or not; if it is, then it is not convenient to increase it, otherwise it is not convenient to decrease it. If the agents want to run no risk we have the following result.

Proposition 1. *If two completely risk-averse agents, i and j, agree on misreporting their valuations v_i and v_j, with $v_i \geq v_j$, they obtain the highest safe gain when agent i declares his valuation v_i and agent j increases his own up to v_i, regardless of the valuation of the other agents.*

Proof. Let b_i and b_j be the declarations of agents i and j, with $b_i \geq b_j$. W.l.o.g. let us suppose that agent i declares $b_i > v_i$ and gets the object, implying that $b_i \geq v_1$. In this case the joint payoff of agents i and j is:

$$v_i - \frac{n-1}{n} b_i + \frac{2}{n}\left[b_i - \frac{1}{n}\left(\sum_{k\neq i,j} v_k + b_i + b_j\right)\right] + \frac{1}{n} b_j$$

On the other hand, if they both declare v_i, independently from getting or not the object, their joint payoff is:

$$\frac{2}{n} v_i + \frac{2}{n}\left[v_1 - \frac{1}{n}\left(\sum_{k\neq i,j} v_k + 2v_i\right)\right]$$

Taking into account the above inequalities on b_i, b_j, v_1, v_i, v_j, it is easy to check that the latter payoff is larger than the first one.

Now, we consider the case in which none of the two agents obtains the object; in this situation their joint variation of payoff is $[(b_i - v_i) + (b_j - v_j)]\left(\frac{n-2}{n^2}\right)$. We may conclude the agents have the possibility of further increase their joint payoff w.r.t. both

declaring v_i, but they may incur the risk of getting the object in case $b_i > v_1$, against the hypothesis of complete risk-aversion. □

If agents i and j make the same declaration v_i according to Proposition 1, at the end of the procedure (besides the monetary compensations among all the agents) one of the following three cases happens:

1. The object is assigned to another agent. The colluding agents know that agent j gains $\frac{n-1}{n^2}(v_i - v_j)$ and agent i loses $\frac{1}{n^2}(v_i - v_j)$. This means that if agent j compensates agent i for his loss then they may share the total gain $\frac{n-2}{n^2}(v_i - v_j)$.
2. The object is assigned to agent i. The situation is similar to that in case 1, but part of the payoff of agent i is represented by the object.
3. The object is assigned to agent j. If agent j gives the object to agent i and receive the monetary amount v_i the two agents are in a situation equivalent to the case 2.

Now, we investigate the credibility of truthful revelation among two completely risk-averse agents.

Proposition 2. *If a completely risk-averse agent i decides to look for an agreement with another agent j with imitation of the highest valuation, he may obtain the highest safe gain truthfully revealing his valuation to agent j.*

Proof. According to Proposition 1, if he declares a false higher valuation $v_i' > v_i$ he may incur the risk of getting the object with a lower payoff. On the other hand, if he makes a lower false declaration $v_i' < v_i$, if $v_i' \geq v_j$ the two agents obtain a lower total payoff, as $\frac{n-2}{n^2}(v_i' - v_j) < \frac{n-2}{n^2}(v_i - v_j)$; moreover the loss and the consequent compensation of agent i reduce, as $\frac{1}{n^2}(v_i' - v_j) < \frac{1}{n^2}(v_i - v_j)$ and a completely risk-averse agent prefers do not incur this risk. □

Agent i always suffers a loss, so his gain originates from the sharing of the total gain, therefore a binding agreement is necessary.

Note that the colluders may know in advance their total gain on the basis of the knowledge of the number of agents, n, and of their own valuations, v_i and v_j; moreover, at the end of the procedure they become conscious of their payoffs if the collusion had not taken place. This provides the colluders a further element for stating how to divide the total gain (for instance they may agree on a division proportional to their payoffs if they did not collude).

To better explain the collusion, let us exemplify it.

Example 2. Referring to the situation in Example 1, let us suppose that agents I and III collude, so the sealed bids of the agents are:

I	II	III	IV
240	192	240	64

As agent III increases his declaration of 160 units, the colluders may compute that agent III will get a gain of 30 units and agent I a loss of 10 units, so agent III refunds the loss of agent I and the two agents share the gain of 20 units.

Now, we want to investigate if the agents may increase their gains enlarging the set of colluders. Let $\mathscr{L} = \{i_1, i_2, ..., i_l\}$ be the set of colluders, with the agents ordered according to weakly decreasing valuations, i.e. $v_{i_1} \geq v_{i_2} \geq ... \geq v_{i_l}$; in this case the optimal strategy is that all the agents declare the highest valuation v_{i_1}. As in the previous case of two colluders, the agreement includes the possibility that the agent that receives the object from the mediator transfers it to one of the colluders with the actual highest valuation with a monetary compensation.

The increase of the declaration of agent i_2, $(v_{i_1} - v_{i_2})$, generates a gain of $\frac{n-1}{n^2}(v_{i_1} - v_{i_2})$ for him and a loss of $\frac{1}{n^2}(v_{i_1} - v_{i_2})$ for the other $l-1$ agents in $\mathscr{L}$, so the final gain for the colluders is $\frac{n-l}{n^2}(v_{i_1} - v_{i_2})$. Repeating the same reasoning for the other agents $i_3, ..., i_l$ we obtain that the total gain for the colluders is $G^{\mathscr{L}} = \frac{n-l}{n^2}\sum_{j=2,...,l}(v_{i_1} - v_{i_j})$.

The gain of the colluders is independent from any fixed (false or true) declaration of the agents not involved in the collusion. Consequently, if two or more groups of colluders form, the variation of the total payoff of each group is not affected by the formation of the others. Of course the final payoff of each agent depends on the declaration of all the agents. Referring to situation in Example 1, let us consider the following four cases:

	Collusion	I	II	III	IV
A	$\varnothing$	84	72	44	40
B	{I, II}	81	81	41	37
C	{III, IV}	83	71	43	43
D	{I, II},{III, IV}	80	80	40	40

It is easy to check that for agents I and II the variation of total payoff from the case A to B, i.e. 6 units, is equal to the variation from the case C to D; analogously for agents III and IV the variation of total payoff from the case A to C, i.e. 2 units, is equal to the variation from the case B to D.

Referring to the situation in Example 1, we may note that the collusion among agents I and II gives them $G^{\{I,II\}} = 6$, the collusion among agents I and IV gives them $G^{\{I,IV\}} = 22$, and the collusion among agents I, II and IV produces the gain $G^{\{I,II,IV\}} = 14$. So, enlarging the set of colluders the gain may increase or decrease. Also the average gain $G^{\mathscr{L}}/|\mathscr{L}|$ may increase or decrease, as $\frac{1}{2}G^{\{I,II\}} < \frac{1}{3}G^{\{I,II,IV\}} < \frac{1}{2}G^{\{I,IV\}}$. Moreover, the gain of a collusion among all the agents is always zero, as $l = n$. So, we may state the following proposition.

Proposition 3. *The gain $G^{\mathscr{L}}$ has no monotonicity property.*

We conclude the section with an example, referring to a hypothetical division of an inheritance among several heirs, that is a classical application of the procedural approach, as pointed out by Brams and Taylor (1996 and 1999) and by Young (1994). In such a situation it is possible that some heirs have a stronger relationship among them, so they decide to collude against the others.

Example 3. Suppose that four heirs Anne (I), Barbara (II), Christine (III) and Daniel (IV) have to divide an estate consisting of an apartment, a studio and a farm. The valuations are as in the following table:

	I	II	III	IV
Apartment	800	880	840	920
Studio	120	96	80	88
Farm	120	144	160	136

The Knaster's procedure assigns to Anne the studio and 166 (with a final payoff of 286), to Barbara 306, to Christine the farm and 136 (296) and to Daniel the apartment but he has to pay 608 (312).

Suppose that Anne, Barbara and Christine are sisters, so that they have an incentive in colluding. So, they declare the highest valuation, among their ones, for each item:

	I	II	III	IV
Apartment	880	880	880	920
Studio	120	120	120	88
Farm	160	160	160	136

W.l.o.g., let us suppose that with false declarations the items are assigned as with actual declarations, but the compensations are different: Anne receives 181, Barbara 301, Christine 141 and Daniel pays 623. So, the total gain for the sisters is 15.

5. Concluding Remarks

In this paper we analyzed the behavior of the agents that misrepresent their valuations in an allocation process, when the Knaster's procedure is the mechanism for assigning objects and compensations to the agents. We may say that the Knaster's procedure is a manipulable mechanism; in fact, each agent has the possibility of a declaration for an object that increases his payoff w.r.t. the true valuation, unless he has the highest valuation and this coincides with that of at least another agent.

On the other hand, the agent does not know the declarations of the other agents, and then he cannot say which declaration is profitable. Up to this point we agree with the opinion of Raiffa (1982), that on the basis of some examples, concludes that "*although one can't really say that the Steinhaus scheme [really the Knaster's procedure] encourages honest valuations, in many situations it may be the pragmatic thing to do. Honesty in this case is the supercautious strategy ... it is also a good strategy against an extreme or naive exaggerator ... (and) it is the easiest and most socially desirable thing to do*."

The property of coalition-strategy-proofness (see Moulin, 1993) does not hold because two agents may improve their payoffs with coordinated false declarations, but again if they do not know the declarations of the other agents they may run a risk (see the proof of Proposition 1). The collusive mechanism studied in this paper is suitable

for completely risk-averse agents that look for a safe gain, obtaining a total gain that does not depend on the declarations of the other agents, so they may know it before the end of the procedure.

On the other hand, the collusive behavior is considered illegal. For instance in case of bankruptcy of a firm, according to the Italian Civil Code (art. 2929), the official receiver may ask for the retraction of the assignment of the goods if a written or tacit collusion is suspected or supposed. Then, an agent may decide to start a collusion with other agents when there exist no moral constraints and some exogenous relationship among the agents (for instance the sisters in Example 3).

Summarizing, the Knaster's procedure has good characteristics from the point of view of individual manipulation, but not so good in case of coalitional manipulation.

Possible further developments are in the direction of introducing a Bayesian game in which the types of the players depend on their valuations and the agents may have beliefs on the types of the other agents and consequently, on their valuations.

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