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The Uncertainty in Voting Power: The Case of the Czech Parliament 1996–2004

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Abstract The main aim of this paper is to study the power of legislators in the Lower House of the Czech Parliament in 1996–2004 with respect to power distribution and its uncertainty. A discrepancy between a-priori computed power indices and outcome of voting leads to necessity to reveal the possible source of uncertainty. This paper studies uncertainty in party loyalty, presence and creation of hidden coalitions and explains the addition of these uncertainty issues to computation of power indices. Recalculated power indices exhibit positive improvement.

Keywords Czech Parliament, power indices, parliamentary voting, uncertainty
JEL classification D71, D72

1. Introduction

A particularly intriguing aspect of legislative studies is the modeling of legislators' decision-making behavior as they are passing legislation. Models based on principles of economics usually describe the behavior of legislators as a political behavior. One of the basic ideas in political science is the measuring influence of political parties in voting legislation. The question of measuring of the power of voting bodies in committees by mathematical means has been studied since 1950*s* (for example in Shapley 1953; Shapley and Shubik 1954; Banzhaf 1965). Mathematicians and economists tried to evaluate the power by indices that could express the true expected power of voting body. These so-called power indices show the ratio of theoretical power of voting body in a committee before the voting occurs. Hence, power indices measure potential influence of voting body members, given a voting rule, as a probability of being decisive. Power indices with necessary mathematical background are described, for example, in Owen (1982a), Turnovec (2003). Historical background of power indices analysis is given in Felsenthal and Machover (2003).

In the real world, the a-priori power of voting is influenced by a chosen voting rule and quota. For example, problems of choosing the optimal quota for the fair voting rules are described in Turnovec (2010). Computations of power indices under the condition of multi-cameral voting system are discussed in Chiriac (2008), Gambarelli and Uristani (2009), and Turnovec (1992).

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The power indices can be influenced not only by voting rules, but also by additional behavioral assumptions about voting configuration formation. Turnovec (2000) in his paper covers three such an assumptions: a-priori union structure, paradox of quarrelling members, and positions of committee members on an ideology space with condition of connected voting configurations. The first, and the most important additional assumption—the assumption of a-priori union structure—was introduced by Owen (1977) and described in Owen(1982a, 1982b). He modified basic Shapley-Shubik and Banzhaf indices approach into models with a-priori union structure. Properties of such models are discussed in Alonso-Meijide et al. (2009).

In the legislative studies of voting, the voting body is usually a political party or coalition of more political parties that vote together in parliamentary voting. Therefore, power indices of parliamentary voting should reveal potential influence of political parties (or coalition of political parties) as a probability of being decisive. In reality, the observed decisiveness of political parties differs from theoretical influence expressed by power indices, as legislators do not vote exactly with their party decisions, or do not vote at all. This legislator's behavior can be caused by various factors. For example, Jackson and Kingdon (1992) argue, that legislators are defecting in their party loyalty, as they are influenced by various political pressure groups. Kau and Rubin (1979) studied how the factor of logrolling and ideology influences US Congressional voting. Importance of deputies' home district, timing and/or seniority factor was discussed by Stratmann (2000). All the mentioned factors could have the influence on the political decisions; however they were not studied together with power indices.

The main aim of this paper is to study the power of legislators in the Lower House of the Czech Parliament in the period of 1996–2004 with respect to power distribution and its uncertainty. More detailed study of the power distribution in the Czech Parliament can be found in Turnovec (1992,1997a, 1997b). Similar studies were done for other countries, for example Chiriac (2008) described the power distribution in the European Parliament and the Council of Ministers; he discusses the distribution of power in the Romanian Parliament; Turnovec (1995) describes political profiles of Visegrad countries.

Main motivation for this paper was the comparison of computed a-priori power indices with voting outcomes obtained from the Lower House of the Czech Parliament in Mielcová (2002). The party success during the electoral period was compared with a-priori power indices. The interesting point was that even ordered parties' voting success indices were not correlated with ordered power indices. This result was quite surprising. However, in social science the classical mathematical tools are usually not enough to predict the behavior of individuals. To measure the obtained party voting success is easy; however it does not model the a-priori power of voting body. The important task is to incorporate uncertainty to the a-priori power indices with respect to previous results.

The mathematical tools dealing with uncertainty are known as the theory of fuzzy-sets (Zimmermann 2001). In order to evaluate the parties' power, the power indices can be recalculated with respect to these mathematical tools. Therefore, the first objective of this paper is to show a discrepancy between a-priori computed power indices and outcome of voting. The second objective is to reveal the possible source of uncertainty and the last objective is to add uncertainty issues to computation of power indices, to recalculate these indices, and to compare them with results of voting.

This paper is organized as follows. Section 2 gives a short description of the Czech Parliament and data used to analyze voting power. Section 3 is devoted to comparison of classical power indices to outcome of voting in the Lower House of the Czech parliament. This part covers review of basic power indices with illustrative examples, the definition of party success coefficient and its comparison. The possible sources of uncertainty are studied in Section 4. These sources of uncertainty are incorporated into the models described in Section 5. Section 6 presenting model tests, results, and discussion is followed by the conclusion in Section 7.

2. The Czech parliamentary system

The Czech Parliament is divided into two chambers—the Chamber of Deputies (the Lower House) and the Senate (the Upper House). The first Parliament was elected before the split in the last Czechoslovak general election held on June 5–6, 1992. Participants elected representatives to the Federal Parliament and to the Czech or Slovak National Council. In December 1992, the Federal Parliament was dissolved and the Czech and Slovak National Councils became the main Parliaments in the Czech and Slovak Republics. From January 1, 1993 the Czech National Council turned into the Lower House of Parliament of the Czech Republic (in Czech "Poslanecká sněmovna Parlamentu České republiky", simply called the 1992 Lower House) and temporarily substituted for the Upper House–Senate. This Lower House performed without the existence of the Senate.

The second Lower House of Parliament was elected for a four-year term in the first Czech elections held on May 31–June 1, 1996. The electoral period of this Lower House, however, was touched by political crises in the winter of 1997. After the government change, the date of a new election was set for June 19–20, 1998. The third Lower House of Parliament was elected for four years, elections were in 2002. Elections to the fourth Lower house were held in 2006, the next Elections are expected to be in 2010.

The Upper House of the Czech Parliament—Senate (in Czech "Senát Parlamentu České republiky")—was first elected in November 1996, thus completing the parliamentary system as required by the Constitution. Members of Senate are elected for six years. Elections to Senate run every two years in one third of electoral regions. There are 81 electoral regions for the Senate elections in the Czech Republic. The first electoral period for members of Senate was two, four or six years depending on their region. The second Senate elections took place in November 1998 in 27 regions; the Upper House elections are regularly every two years.

The deputies of the Chamber of Deputies (the Lower House) are elected on the basis of universal, equal, and direct secret ballot voting according to the principles of proportional representation. All 200 members of the Chamber of Deputies are elected for a four-year period.

Voting in the Lower House may be public, usually through the use of voting equipment and raising hands, or secret, through the use of voting ballots. Voting on legislation is always public.

Deputies use voting equipment to announce their presence in the meeting room, and their presence is double checked before each voting. A quorum is constituted when at least one third of all Deputies are present. Lower House members vote for or against a proposal through the use of voting equipment. To be passed, proposals usually need a simple majority of votes equal to one half of all present legislators. Exceptions to this rule are named in Act No. 90/1995:

"(3) *Approving the Constitutional Act, approving of international agreements on human rights and basic freedoms shall require the approval of a three fifths majority of all Deputies.*

(4) *Adopting declarations of a state of war and adopting resolutions expressing agreement with the presence of foreign troops in the territory of the Czech Republic shall require the agreement of a simple majority of all Deputies. The agreement of a simple majority of all Deputies shall also be required when the Chamber of Deputies votes on draft acts which were rejected by the Senate, when the Chamber of Deputies votes on acts which were returned by the President of the Republic and when the Chamber of Deputies votes on non-confidence in the Government.*"[1]

3. Comparison of a-priori voting power with results of voting

The voting power is traditionally expressed by power indices. For purposes of this analysis I decided to use two of them—the Shapley-Shubik power index and the Banzhaf power index. The indices are described in the next part of this article, followed by illustrative examples. The detailed description of power indices can be found in Owen (1982a).

3.1 Power indices

Originally, power indices were defined in the framework of cooperative game models. The cooperative possibilities of voting body can be described by a characteristic function v that assigns a number $v(S)$ to every coalition S. Here $v(S)$ is called the worth of coalition S, and it represents the total amount of transferable utility that the members of S could earn without any help from the players outside S. A characteristic function can also be called a game in coalitional form or a coalitional game (Myerson 1991). The characteristic function of voting can reach values 0 or 1. Thus, the characteristic function, as well as the respective coalitional game can be shortly described by a quota q and n members' weights $w_1, w_2, \ldots, w_n$ as a set $\{q; w_1, w_2, \ldots, w_n\}$. A power index of such a coalitional game $\{q; w_1, w_2, \ldots, w_n\}$ would represent a reasonable expectation of the decisional power share of various players in the game, given by ability to create

[1] Basic information on the Czech Parliamentary system can be found at the official web site of the Lower House of the Czech Parliament, http://www.psp.cz, and at the official web site of the Upper House of the Czech Parliament, http://www.senat.cz.

winning coalitions. Let $\pi_i(q;\mathbf{w})$ denotes the share of power that power index grants to the i-th member of a committee with allocation $\mathbf{w}$ and quota q. In the legislator's voting, each member, or so-called player, represents one political party.

In order to measure the power of players in voting, several power indices were created. The oldest power index—called the Shapley-Shubik power index (presented in Shapley and Shubik 1954)—is widely used in a game theory to evaluate the power of cooperative games. The Banzhaf power index is more proper for normative questions (Banzhaf 1965); the Holler-Packel power index (Holler and Packel 1983) together with Schmeidler power index (Schmeidler 1969) are more suitable for evaluation of special electoral schemes and they are used in the fair division theory (Owen 1982a). This analysis was based on two power indices—the Shapley-Shubik power index and the normalized Banzhaf power index; both power indices are monotone, stable and commonly used to evaluate the power of voting. The description of these two power indices together with small examples is done in the next part; this description of power indices is based on Turnovec (2003, 2007).

The *Shapley-Shubik power index* is derived from the model of bargaining, where the power index takes all possible preferences' allocations. This power index is given by Shapley value reduced to simple game approach (Shapley 1953). The Shapley-Shubik power index is given as

$$\pi_i^{SS}(q;\mathbf{w}) = \sum_{S\in W(i)} \frac{(s-1)!(n-s)!}{n!},$$

where summation is taken over the set $W(i)$, defined as the set of the vulnerable coalitions for which a player i is essential (that means that S is winning such that $S\backslash\{i\}$ is not winning); s is the cardinality of S.

Example 1. A committee is composed of 10 members grouped into three political parties, A, B, C with numbers of members 5, 3, 2, respectively. The quota to simple majority voting was expected to be 6. Value of denominator in preceding relation is equal to $n! = 3! = 6$. There are three coalitions with player A as an essential player: $\{AB, AC, ABC\}$, and one coalition with player B or C as an essential player: $\{AB\}$, $\{AC\}$. The Shapley-Shubik power index can be computed as:

$$\pi_A^{SS} = \frac{(2-1)!(3-2)!}{6} + \frac{(2-1)!(3-2)!}{6} + \frac{(3-1)!(3-3)!}{6} = \frac{2}{3},$$

$$\pi_B^{SS} = \pi_C^{SS} = \frac{(2-1)!(3-2)!}{6} = \frac{1}{6}.$$

To compute the Shapley-Shubik index using different approach, we need to list all possible preferences' allocations and to indicate essential players (or so-called pivotal players). Thus, the game is of the form: $\{q=6; w_A=5, w_B=3, w_C=2\}$. The list of all possible preferences' allocations (that means $n! = 3! = 6$ permutations) ranked from left to right as from approval to disapproval, essential voters are underlined: $\{A\underline{B}C, A\underline{C}B, B\underline{A}C, BC\underline{A}, C\underline{A}B, CB\underline{A}\}$. Player A is essential four times out of six, the power index $\pi_A^{SS} = \frac{4}{6} = \frac{2}{3}$. Similarly, $\pi_B^{SS} = \frac{1}{6}, \pi_C^{SS} = \frac{1}{6}$.

The *normalized Banzhaf power index* (called also relative Penrose-Banzhaf power index, Banzhaf 1965; Owen 1982b, Felsenthal and Machover 2003, Turnovec 2010) is based on the idea of all possible situations, when the member is critical for the given coalition—on idea of swings. The originally proposed absolute Penrose-Banzhaf index is defined as the ratio of number of swings of i-th member to the number of all coalitions of i-th member. While the absolute Penrose-Banzhaf power index is based on probabilistic theory, relative Penrose-Banzhaf (normalized Banzhaf) index answers simply the question what is the voter i's share in all-possible swings. The normalized Banzhaf index is defined as the ratio of the i-th player's swings number θ_i to the total number swings of all players' swings. A swing of player i is a pair $(S, S\backslash\{i\})$; S is winning and $S\backslash\{i\}$ is not winning coalition. The index is given as:

$$\pi_i^B = \frac{\theta_i}{\sum\limits_{j\in N}\theta_j}.$$

The sum of normalized Banzhaf indices over all players gives one. For brevity, the word "normalized" is omitted in this index name from now on.

Example 2. Let's expect the same example as above, which means the voting game of the form: $\{q=6; w_A=5, w_A=3, w_A=2\}$. To compute the Banzhaf power index we need to list all winning coalitions. The list of all possible coalitions, winning coalitions are underlined: $\{\emptyset, A, B, C, \underline{AB}, \underline{AC}, BC, \underline{ABC}\}$. The set of all winning coalitions, swing players are underlined: $\{\underline{AB}, \underline{AC}, \underline{A}BC\}$. Player A is a swing player three times, while players B and C are swing players once. Thus, the Banzhaf power index is $\pi_A^B = \frac{3}{5}$, $\pi_B^B = \frac{1}{5}$, $\pi_c^B = \frac{1}{5}$.

For the case of a-priori coalition structure, declared for example by coalition treaty, the correct way to evaluate the power is to compute so-called *power index of the committee with a priori union structure*. The idea of power index with a-priori coalition structure was introduced by Owen (1977). To explain the idea, we expect that n is the number of voting body members with respective weights $w_1, w_2, \ldots, w_n$ voting on proposal using the majority rule with quota q—voting game $\{q; w_1, w_2, \ldots, w_n\}$. Denote $\pi_i(q; w_1, w_2, \ldots, w_n)$ is the power index of i-th member of this voting game. Let's consider that members are creating m unions (coalitions) with weights $t_1, t_2, \ldots, t_m$. Every union is composed of several members; each of them consists of at least one member. Denote $\mathbf{w}^j$ be the set of members in j-th union.

Let's consider two-level process: the first level inter-union committee game $\{q; t_1, t_2, \ldots, t_n\} = \{q; \mathbf{t}\}$ with the power of j-th coalition $\pi_j(q; \mathbf{t})$; the m internal union second level subgames $\{q; t_1, t_2, \ldots, t_{j-1}, \mathbf{w}^j, t_{j+1}, \ldots, t_m\} = \{q; \mathbf{t}, \mathbf{w}^j\}$ with the power index of i-th member of the j-th coalition $\pi_{i,j}(q; \mathbf{t}, \mathbf{w}^j)$. Then the power index of the member i of the union j in the committee with a priori union structure is

$$\pi_i(q; \mathbf{w}, \mathbf{t}) = \pi_j(q; \mathbf{t}).\frac{\pi_{ij}(q; \mathbf{t}, \mathbf{w}^j)}{\sum_{k\in K_j}\pi_{k,j}(q; \mathbf{t}, \mathbf{w}^j)},$$

where k is going through the set K_j containing all members in union j.

Example 3. Let's expect the same example as before, that means the game of the form: $\{q = 6; w_A = 5, w_B = 3, w_C = 2\}$; members A and B created coalition. Let's compute the Shapley-Shubik power index with given a-priori coalition function. Thus, there are two coalitions, $t_1 = \{A, B\}$ with weight 8 and $t_2 = \{C\}$ with weight 2. It is apparent that the power of the coalition $\{A, B\}$ is $\pi_1^{SS}[q; \mathbf{t}] = 1$ while the power of the coalition $\{C\}$ is $\pi_2^{SS}[q; \mathbf{t}] = 0$. In this game, the second level sub-game's Shapley-Shubik power indices are $\pi_A^{SS} = \frac{3}{6}$, $\pi_B^{SS} = \frac{1}{6}$. Then the Shapley-Shubik power indices of members in the committee with a priori union structure are:

$$\begin{aligned}
\pi_A^{SS}(q; \mathbf{w}, \mathbf{t}) &= \pi_1^{SS}(q; \mathbf{t}) \cdot \frac{\pi_{A,1}(q; \mathbf{t}, \mathbf{w}^j)}{\pi_{A,1}(q; \mathbf{t}, \mathbf{w}^j) + \pi_{B,1}(q; \mathbf{t}, \mathbf{w}^j)} = 1 \cdot \frac{\frac{3}{6}}{\frac{3}{6} + \frac{1}{6}} = \frac{3}{4}, \\
\pi_B^{SS}(q; \mathbf{w}, \mathbf{t}) &= \pi_1^{SS}(q; \mathbf{t}) \cdot \frac{\pi_{B,1}(q; \mathbf{t}, \mathbf{w}^j)}{\pi_{A,1}(q; \mathbf{t}, \mathbf{w}^j) + \pi_{B,1}(q; \mathbf{t}, \mathbf{w}^j)} = 1 \cdot \frac{\frac{1}{6}}{\frac{3}{6} + \frac{1}{6}} = \frac{1}{4}, \\
\pi_C^{SS}(q; \mathbf{w}, \mathbf{t}) &= 0.
\end{aligned}$$

3.2 Coefficient of voting success

To evaluate the party success after voting, the index of party success can be constructed by comparing party decision with the outcome of voting. The coefficient of party success can evaluate the party success during a parliamentary period. The party A success index is defined as the ratio of decisions of the Lower House that were the same as the party A decisions to all decisions during the parliamentary period:

$$I_{success}^{A} = \frac{\text{number of party } A \text{ decisions identical with parliamentary decisions}}{\text{number of parliamentary decisions}}$$

The party decision is derived from the votes of party members using simple majority rule. The coefficient of party success can reach the values from the interval $\langle 0, 1 \rangle$; the higher the coefficient, the higher ratio of party decisions was the same as the whole voting body decision. The coefficient of voting success is influenced by the vote other members of parliament. Small parties with low power index may have a large success if they vote following the majority of votes. That means, the index of voting success should better described power of political parties with a-priori coalition structure, however, the index is taken as a rough political parties' success measure.

3.3 Comparison of power index with parties' voting success

The following analysis is based on data collected from Lower House roll call voting in 1996–2004 period. This time interval covers almost three parliamentary periods: the period after 1996 elections, the period after 1998 elections and partially the period after 2002 elections. The uncertainty incorporated into a-priori voting power should be based on the data from the preceding period. Voting results are accessible to the public and the media upon request.[2]

[2] Information about voting in the Lower House is accessible at the official Parliamentary Internet domain www.psp.cz.

Data are collected with respect to votes in voting vectors. The voting vectors are set with respect to Mielcova (2002a)—one voting vector contains voting outcomes for one bill of all 200 members. The outcome of every vote for every member can be one number of the set $\{0,1,2,3\}$, which indicates member preferences: 0 –“no”, 1 –“yes”, 2 –“present, abstain”, 3 –“absent”. Every bill to be passed needs at least as many “yes” votes as quota. Quota is based on the sum of all present legislators, which means on the sum of all legislators with the voting outcome from the set $\{0,1,2\}$. Hence outcome “present, abstain” serves as “no” outcome so in this analysis this outcome is reclassified as “no” outcome. Data for the first, 1996–1998 period covered 4741 voting vectors. Data for the 1998–2002 and 2002–2004 periods covered 14081 and 4794 voting vectors, respectively.

During the studied period, there were seven political parties active in the Czech Parliament. The names of political parties, followed by the abbreviation used in this paper and by the electoral years, which indicates when parties were functioning in the Lower House are given in Table 1.

Table 1. List of parties

Civic Democratic Party	ODS	1996, 1998, 2002
Czech Social Democratic Party	CSSD	1996, 1998, 2002
Christian and Democratic Union-Czechoslovak People’s Party	KDU-CSL	1996, 1998, 2002
Czech and Moravian Communist Party	KSCM	1996, 1998, 2002
Freedom Union*	US	1996, 1998, 2002
Civic Democratic Alliance	ODA	1992, 1996
Association for Republic-Czechoslovak Republican Party	SPR-RSC	1996

* After the 2002 elections as a coalition of two political parties—Freedom Union and Democratic Union.

The power indices for the period of 1996–2004 can be divided into three groups with respect to the Elections to the Lower House of the Czech Parliament.

Parliamentary period 1996–1998

After the 1996 elections, there were 6 political parties operating in the Czech Parliament. However, during the parliamentary period, there was one substation change in party structure significantly affecting power distribution. Power indices for this period are in Table 2.

Table 2. Power indices of Lower House political parties during 1996–1998

Session	Power Index	ODS	CSSD	KDU-CSL	ODA	KSCM	SPR-RSC	US	no p.a.
1–18	Shapley-Shubik	0.333	0.267	0.100	0.033	0.167	0.100	–	–
	Banzhaf	0.321	0.250	0.107	0.036	0.178	0.107	–	–
19–26	Shapley-Shubik	0.171	0.322	0.080	0.070	0.080	0.080	0.171	3×0.005
	Banzhaf	0.169	0.319	0.081	0.071	0.091	0.081	0.169	3×0.006

Note: There were three members without party affiliation from 19th to 26th parliamentary sessions. Computed power indices of these members are given in column marked “no p.a.”.

Moreover, at the beginning of the parliamentary period, three political parties—ODS, ODA, and KDU-CSL—formed coalition in order to set up the Government. This coalition was affecting power indices. For the period after the 18th parliamentary session, the newly created political party—the Freedom Union—would behave as coalitional parties. The power indices with a-priori coalitional structure for this period are given in Table 3.

Table 3. Power indices with a-priori coalitional structure for Lower House political parties during 1996–1998

Session	Power Index	ODS	CSSD	KDU-CSL	ODA	KSCM	SPR-RSC	US	no p.a.
1–18	Shapley-Shubik	0.357	0.167	0.107	0.035	0.167	0.167	–	–
	Banzhaf	0.357	0.167	0.107	0.035	0.167	0.167	–	–
19–26	Shapley-Shubik	0.223	0.076	0.105	0.091	0.076	0.076	0.223	3×0.043
	Banzhaf	0.245	0.060	0.116	0.102	0.060	0.060	0.243	3×0.036

Note: The coalition is created by political parties ODS, ODA, and KDU-CSL during the sessions 1–18 and by the political parties ODS, ODA, US and KDU-CSL during the sessions 19–26. Moreover, there were three members without party affiliation from 19th to 26th parliamentary sessions. Computed power indices of these members are given in column marked “no p.a.”.

Coefficients of success for all political parties for the period before and after the creation of Freedom Union, and for the whole period are given in the Table 4. Coefficients of party success are quite high; it is caused by the fact that huge amount of votes was done unanimously across political parties.

Table 4. Coefficient of success of Lower House political parties during 1996–1998

Session	ODS	KDU-CSL	US	CSSD	KSCM	ODA	SPR-RSC
1–18	0.853	0.865	–	0.697	0.546	0.839	0.303
19–26	0.854	0.840	0.876	0.659	0.503	0.793	0.459
Whole period	0.854	0.858	0.876	0.686	0.533	0.826	0.348

Table 5. Correlation coefficients of party success with power indices

Session		Pearson’s corr. coeff. Index of Success	Spearman’s corr. coeff. Index of Success
1–18	Shapley-Shubik	0.173	0
	Banzhaf	0.147	0
	S-S with coalition	0.007	−0.213
	B with coalition	0.007	−0.213
19–26	Shapley-Shubik	0.148	0.318
	Banzhaf	0.123	0.273
	S-S with coalition	0.702	0.954**
	B with coalition	0.772*	0.954**

Note: ** denotes significance at the 0.01 level (2-tailed), * denotes significance at the 0.05 level (2-tailed).

To compare the linear dependence of success coefficient and the power index, the Pearson and Spearman's correlation coefficients with its statistical significance were computed (Myers and Well 2003). Results of computations are given in Table 5.

From the results it's apparent, that common power indices are not declaring future success in voting. In both periods the coefficients of success and common power indices were not statistically significant. However, the power indices with a-priori coalitional structure are better describing the behavior of legislators, as it happened in the second part of this parliamentary period, when the correlation coefficient is high and statistically significant.

Parliamentary period 1998–2002

After 1998 Elections, there were five political parties operating in the Lower House of the Czech Parliament. During the period, there were only two switches in party affiliation; only one change was causing a change in power. The power indices for this period are given in Table 6.

Table 6. Power indices of Lower House political parties during 1998–2002

Date	Power Index	ODS	KDU-CSL	US	CSSD	KSCM	no p.a.
15.7.1998	Shapley-Shubik	0.300	0.133	0.133	0.300	0.133	–
	Banzhaf	0.286	0.143	0.143	0.286	0.143	–
16.10.2000	Shapley-Shubik	0.291	0.124	0.124	0.307	0.141	2×0.007
	Banzhaf	0.274	0.133	0.133	0.292	0.150	2×0.009

Note: After the 10/16/2000, there were two parliamentary members without party affiliation. Computed power indices of these members are given in column marked "no p.a.".

At the beginning of this parliamentary period, two political parties—CSSD and ODS—signed so-called opposition treaty. This treaty helped to create a government to ODS with support of CSSD, even though CSSD was in opposition to ODS. The power indices with this a-priori coalitional structure are given in Table 7. The coefficients of success, computed for all political parties the whole period are given in Table 8.

Table 7. Power indices with a-priori coalitional structure for Lower House political parties during 1998–2002

Power Index	ODS	CSSD	KDU-CSL	KSCM	US
Shapley-Shubik	0.5	0.5	0	0	0
Banzhaf	0.5	0.5	0	0	0

Note: The a-priori coalition is created by the political parties ODS and CSSD—these political parties signed so-called opposition treaty.

Correlation coefficient between all power indices and coefficient of success is always the same, 0.378. Also the ranks of power indices with and without a-priori coalitional structure are the same. The correlation coefficient between ranks of power indices and voting success coefficient is the same as the correlation coefficient

Table 8. Coefficient of success of Lower House political parties during 1998–2002

Comments	ODS	KDU-CSL	US	CSSD	KSCM
Whole period	0.777	0.833	0.813	0.874	0.675

between ranks of power indices with a-priori coalitional structure and voting success coefficient. This correlation coefficient is 0.289. Both correlation coefficients are not statistically significant. For this parliamentary period, the a-priori power indices were not describing party success.

Parliamentary period 2002–2004

During the parliamentary period after the 2002 Elections, there were five political parties operating in the Lower House of the Czech Parliament. The used data set is not complete, it covers 26 parliamentary sessions.

After the 2002 Elections, the government was created by CSSD with support of KDU-CSL and US, even though KDU-CSL and US are commonly in opposition with CSSD. Therefore, the power indices with a-priori coalition structure are based on the coalition of these three political parties.

Table 9. Power indices and coefficient of success of Lower House political parties during period 2002–2004

Ranks	ODS	CSSD	KDU-CSL	KSCM	US
Shapley-Shubik	0.233	0.400	0.067	0.233	0.067
Banzhaf	0.231	0.385	0.077	0.231	0.077
S-S with coalition	0	0.749	0.125	0	0.125
Banzhaf with coalition	0	0.714	0.143	0	0.143
Coefficient of Success	0.596	0.928	0.904	0.697	0.203

Table 10. Correlation coefficients of party success with power indices of Lower House political parties during 2002–2004

	Index of Success
Shapley-Shubik	0.495
Banzhaf	0.495
S-S with coalition	0.458
Banzhaf with coalition	0.446

The ranks of Shapley-Shubik and the Banzhaf power indices are the same. The correlation coefficient between ranks of success coefficient and power indices (both common power indices and power indices with coalitional structure) is -0.527 and is not statistically significant. Correlations of common power indices with coefficient of party success are given in Table 10, none of them are statistically significant. In this case the computed power indices are not describing future success of political parties in voting.

4. Possible Sources of Uncertainty in Voting Power

The comparison of power indices and coefficient of party success revealed that power indices based solely on the number of political parties' members are not sufficient indicators of future voting success. The idea of power indices is very sophisticated and is clearly analyzing theoretical situation. However, if a model has to describe a real situation, the data entering into calculations of power indices could be adjusted to fit the reality, for example with respect to preceding periods' results. This part of the paper is devoted to looking for such an uncertainty in the data of votes in the Lower House of the Czech Parliament. Namely, to uncertainty that can be revealed from the rough data more than from the political analyst reviews—to party loyalty, legislator's presence at parliamentary sessions and possible existence of "hidden" coalitions.

4.1 Loyalty

Loyalty of party members can be easily expressed as the willingness to vote identically with colleagues from the same political party. Small loyalty can affect the outcome of voting. If incorporating into power index, it could change its value and better describe the power distribution in voting body, as given in the next illustrative example.

Example 4. Let's expect the game of the form: $\{q=6; w_A=6, w_B=3, w_C=2\}$. Both Shapley-Shubik and Banzhaf power indices are the same—$(1,0,0)$ for parties A, B and C, respectively. If we knew, for example, that members of party A are voting the same only in 80% of cases, members of party B in 60% of cases, while members of party C are voting always together, then we could compute the power index with expected values of weights—that means weights $(4.8, 1.8, 2)$. The values would be $\left[\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right]$ for Shapley-Shubik power index and $\left[\frac{3}{5}, \frac{1}{5}, \frac{1}{5}\right]$ for the Banzhaf power index.

Legislator's i loyalty index for given period can be easily computed for every member of political party as the ratio of legislator's votes according to the party votes over all votes through the period:

$$I^i_{loyalty} = \frac{\text{number of } i\text{'s votes identical with party votes}}{\text{number of parliamentary votes}}$$

This variable can reach the value from the interval $\langle 0, 1\rangle$, the higher value—the higher loyalty of the legislator. For the political party as a whole, the party loyalty index can be expressed as the average value of all party's members. Party loyalty index can reach the value from the interval $\langle 0.5, 1\rangle$ because at least one half of present legislators of one political party are voting accordingly. Party loyalty index values for the data from the Lower house of the Czech parliament are given in Table 11.

The party loyalty indices are very similar for all political parties. They can affect the computed power indices very slightly. For example, comparison of original power

Table 11. Party loyalty indices for 1996-2004 Lower House of the Czech Parliament

Period	ODS	CSSD	KDU-CSL	KSCM	US	ODA	SPR-RSC
1996–1998	0.866	0.824	0.876	0.880	0.831	0.866	0.967
1998–2002	0.899	0.917	0.895	0.920	0.896	–	–
2002–2004	0.888	0.928	0.928	0.912	0.918	–	–

indices and adjusted power indices at the beginning of each parliamentary period is given in Table 12. Adjustments were made for the respectively period; this approach is enough in order to reveal possible source of uncertainty. However, for calculation of a-priori power indices we have to adjust data accordingly to the preceding period.

Table 12. Comparison of power indices of Lower House of the Czech Parliament

Period	Power Index	ODS	CSSD	KDU-CSL	ODA	KSCM	SPR-RSC	US
1996–1998	Shapley-Shubik	0.333	0.267	0.100	0.033	0.167	0.100	–
	Banzhaf	0.321	0.250	0.107	0.036	0.178	0.107	–
	Adjusted S-S	0.400	0.250	0.100	0.050	0.100	0.100	–
	Adjusted Banzhaf	0.380	0.260	0.100	0.060	0.100	0.100	–
1998–2002	Shapley-Shubik	0.300	0.300	0.133	–	0.133	–	0.133
	Banzhaf	0.286	0.286	0.143	–	0.143	–	0.143
	Adjusted S-S	0.200	0.450	0.117	–	0.117	–	0.117
	Adjusted Banzhaf	0.200	0.440	0.120	–	0.120	–	0.120
2002–2004	Shapley-Shubik	0.233	0.400	0.067	–	0.233	–	0.067
	Banzhaf	0.231	0.385	0.077	–	0.231	–	0.077
	Adjusted S-S	0.250	0.417	0.083	–	0.250	–	–
	Adjusted Banzhaf	0.250	0.417	0.083	–	0.250	–	–

The ranks of adjusted power indices are the same as ranks of original power indices. Therefore, the legislator's loyalty is not the only factor that influences the voting power in the Czech Parliament. However, this factor could be incorporated into adjustment of entering data into power index computation.

4.2 Legislator's presence at parliamentary sessions

Another factor, that can influence the power of legislators in voting, and that could be easily incorporated into the computation of power indices is the legislator's presence at sessions. This factor can influence the outcome of voting by two ways—absent legislators are not voting, and they are influencing the quota of voting, as well, as it's illustrated in the next example.

Example 5. Let's expect the game of the form: $\{q = 6; w_A = 6, w_B = 3, w_C = 2\}$. Both Shapley-Shubik and Banzhaf power indices are the same—$(1,0,0)$ for parties A,

B and C, respectively. If we knew, for example, that 3 members of party A would be present at voting with probability 0.5, then the Shapley-Shubik power index is $(1,0,0)$ with probability 0.5 for original game, and $\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right]$ with probability 0.5 for game of the form of $\{q=4; w_A=3, w_B=3, w_C=2\}$. Expected value of the Shapley-Shubik power index would be:

$$E(\pi^{ss}) = 0.5 \times [1,0,0] + 0.5 \times \left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right] = \left[\frac{4}{6},\frac{1}{6},\frac{1}{6}\right]$$

Legislator's i presence index for given period can be easily computed for every member of political party as the ratio of i's presences in voting over all votes through the period:

$$I^i_{presence} = \frac{\text{number of } i\text{'s votes}}{\text{number of parliamentary votes}}$$

This variable can reach the value from the interval $\langle 0,1 \rangle$, the higher value—the higher presence of the legislator in voting. For the political party as a whole, the party presence index can be expressed as the average value of all party members' indices. This index can reach the value from the interval $\langle 0,1 \rangle$. Party presence index values for the data from the Lower house of the Czech parliament are given in Table 13.

Table 13. Party presence indices for 1996-2004 Lower House of the Czech Parliament

Period	ODS	CSSD	KDU-CSL	KSCM	US	ODA	SPR-RSC
1996–1998	0.991	0.983	0.984	0.965	0.993	0.964	0.999
1998–2002	0.852	0.851	0.815	0.859	0.745	–	–
2002–2004	0.850	0.867	0.880	0.884	0.771	–	–

The party presence indices, the same as party loyalty indices are similar for all political parties. They can affect the computed power indices very slightly. Moreover, there can exist a correlation between the loyalty and absence coefficient. However, in preceding periods, the political parties punished their members with low loyalty and presence by no access to possible reelection (Mielcova 2002b).

Thus, we can expect that these variables have a small influence on possible voting outcome. For example, comparison of original power indices and adjusted power indices at the beginning of each parliamentary period is given in Table 14. It contains computed power indices for the situation when both party loyalty and presence are incorporated into computation (denoted as L*P adjustment).

The adjusted power indices are similar to original power indices. They rank is almost the same. Correlation coefficients of power indices with party success are not much different comparing to common power indices. Therefore, the legislator's loyalty and party presence are not factors influencing the voting power, but these factors could be incorporated into adjustment of entering data into power index computation.

Table 14. Comparison of power indices of Lower House of the Czech Parliament

Period	Power Index	ODS	CSSD	KDU-CSL	ODA	KSCM	SPR-RSC	US
1996–1998	Shapley-Shubik	0.333	0.267	0.100	0.033	0.167	0.100	–
	Banzhaf	0.321	0.250	0.107	0.036	0.178	0.107	–
	Adjusted S-S	0.367	0.233	0.100	0.067	0.133	0.100	–
	Adjusted Banzhaf	0.357	0.214	0.107	0.071	0.143	0.107	–
	L*P adjusted S-S	0.400	0.250	0.100	0.050	0.100	0.100	–
	L*P adjusted Banzhaf	0.380	0.260	0.100	0.060	0.100	0.100	–
1998–2002	Shapley-Shubik	0.300	0.300	0.133	–	0.133	–	0.133
	Banzhaf	0.286	0.286	0.143	–	0.143	–	0.143
	Adjusted S-S	0.350	0.350	0.100	–	0.100	–	0.100
	Adjusted Banzhaf	0.364	0.364	0.090	–	0.090	–	0.090
	L*P adjusted S-S	0.300	0.550	0.050	–	0.050	–	0.050
	L*P adjusted Banzhaf	0.368	0.474	0.053	–	0.053	–	0.053
2002–2004	Shapley-Shubik	0.233	0.400	0.067	–	0.233	–	0.067
	Banzhaf	0.231	0.385	0.077	–	0.231	–	0.077
	Adjusted S-S	0.283	0.367	0.117	–	0.200	–	0.033
	Adjusted Banzhaf	0.280	0.360	0.120	–	0.200	–	0.040
	L*P adjusted S-S	0.250	0.333	0.167	–	0.167	–	0.083
	L*P adjusted Banzhaf	0.250	0.333	0.167	–	0.167	–	0.083

Note: Adjustments were made for party presence index, and for both party loyalty and presence index (L*P).

4.3 Hidden coalitions

Another factor that can influence the power distribution in legislative process is the existence of hidden coalitions and oppositions. The existence of declared coalitions is covered into the power index with a-priori coalitional structure. However, sometimes some unexpected coalitions and oppositions emerge and their existence influences the power index, as is apparent in the next example.

Example 6. Let's expect the game of the form: $\{q = 6; w_A = 5, w_B = 3, w_C = 2\}$. The Banzhaf power index of this game is: $\pi_A^B = \frac{3}{5}$, $\pi_B^B = \frac{1}{5}$, $\pi_c^B = \frac{1}{5}$. Now expect, that players A and C will never vote accordingly. To compute the Banzhaf power index we need to list all winning coalitions. All possible coalitions with winning coalitions underlined are: $\{\emptyset, A, B, C, \underline{AB}, \underline{AC}, BC, \underline{ABC}\}$. The set of all winning coalitions with swing players underlined is: $\{\underline{AB}, \underline{A}\underline{C}, \underline{A}BC\}$. However, coalitions AC and ABC can never occur, because players A and C are "not playing together". Thus the Banzhaf power index of such a situation is: $\pi_A^B = \frac{1}{2}$, $\pi_B^B = \frac{1}{2}$, $\pi_c^B = 0$.

Possible hidden political parties' coalitions and oppositions can be revealed by correlations of voting outcomes through the whole period. Correlation coefficients between votes of political parties are given in Table 15; all correlation coefficients are statistically significant at the 0.01 level.

In order to incorporate the hidden coalition structure we have it is important to calculate the measure of voting similarities between political parties. The outcome of each voting is a number 0 or 1; therefore the measure of voting similarities can be expressed as the ratio of the number of the same votes of two political parties over the number of all votes. Computed measures of parties' similarities are in Table 16.

Table 15. Votes correlation coefficients Lower House of the Czech Parliament

1998–2002	ODS	CSSD	KSCM	KDU-CSL	US	Result
ODS	1					
CSSD	0.237	1				
KSCM	−0.027	0.357	1			
KDU-CSL	0.413	0.421	0.123	1		
US	0.481	0.362	0.072	0.719	1	
Result	0.508	0.726	0.301	0.632	0.586	1
2002–2004	ODS	CSSD	KSCM	KDU-CSL	US	Result
ODS	1					
CSSD	−0.028	1				
KSCM	0.150	0.169	1			
KDU-CSL	0.030	0.863	0.118	1		
US	0.407	−0.660	−0.093	−0.714	1	
Result	0.122	0.844	0.309	0.792	−0.544	1

Note: These correlation coefficients were computed using computed voting outcome of the respective political parties for all votes.

Table 16. Measure of voting similarities for political parties in Lower House of the Czech Parliament

1998–2002	ODS	CSSD	KSCM	KDU-CSL	US
ODS	1				
CSSD	0.654	1			
KSCM	0.529	0.700	1		
KDU-CSL	0.738	0.735	0.596	1	
US	0.770	0.710	0.576	0.875	1
2002–2004	ODS	CSSD	KSCM	KDU-CSL	US
ODS	1				
CSSD	0.527	1			
KSCM	0.629	0.637	1		
KDU-CSL	0.551	0.936	0.612	1	
US	0.588	0.157	0.340	0.139	1

Note: For each pair of political parties, this measure was obtained as ratio of votes when both political parties voted accordingly over number of all votes.

Measures of voting similarities have higher variation than that of party loyalty index and the party presence index and that measures can have higher influence on power distribution than the index of loyalty and presence.

5. Incorporation of uncertainty into computation of power indices

In the above analysis, the possible measurable sources of uncertainty only slightly influenced the value of power indices. The next task is to incorporate all of them into the computation of the power indices, such that the power index would be adjusted with respect to preceding period. The mathematical tool that deals with uncertainty is the theory of fuzzy sets.

In the book “Fuzzy set theory and its applications” Zimmermann (2001) discussed causes of uncertainty. He listed six possible causes of uncertainty in process: lack of information, abundance of information, conflicting evidence, ambiguity, measurement, and belief. He discussed the uncertainty modeling and the probability of fuzzy events as fuzzy sets. The uncertainty in distribution of voting power in legislative process is partly based on each of the item from this list.

The main idea how to incorporate this uncertainty is to expect that each political party is a fuzzy set with legislators its members defined by a membership function:

A *fuzzy set* $\tilde{A} = (U, \mu_{\tilde{A}})$ on universe (domain) U is a set defined by the membership function $\mu_{\tilde{A}}$ which is a mapping from the universe U into the unit interval: $\mu_{\tilde{A}} : U \to \langle 0, 1\rangle$; $F(U)$ denotes the set of all fuzzy sets on U.

If the value of the membership function $\mu_{\tilde{A}}(x)$, called the membership degree (or grade), equals one, x belongs completely to the fuzzy set. If it equals zero, x does not belong to the set. If the membership degree is between 0 and 1, x is a partial member of the fuzzy set:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \text{ is not member of } \tilde{A} \\ (0,1) & x \text{ is a partial member of } \tilde{A} \\ 1 & x \text{ is a full member of } \tilde{A} \end{cases}$$

Ordinary (non-fuzzy) sets are usually called *crisp (or hard) sets*. *Support*, *core* and α-*cut* are *crisp sets* obtained from a fuzzy set by selecting its elements whose membership degrees satisfy certain conditions.

Definition 1. *The support of a fuzzy set $\tilde{A} = (U, \mu_{\tilde{A}})$ is the crisp subset of U whose elements all have nonzero membership grades: $Supp(\tilde{A}) = \{x \in U; \mu_{\tilde{A}}(x) > 0\}$.*

Definition 2. *The core of a fuzzy set $\tilde{A} = (U, \mu_{\tilde{A}})$ is a crisp subset of U consisting of all elements with membership grades equal to one: $Core(\tilde{A}) = \{x \in U; \mu_{\tilde{A}}(x) = 1\}$.*

Definition 3. *The α-cut $[A]_\alpha$ of a fuzzy set $\tilde{A} = (U, \mu_{\tilde{A}})$ is the crisp subset of U whose elements all have membership grades greater than or equal to α: $[A]_\alpha = \{x \in U; \mu_{\tilde{A}}(x) \geq \alpha\}$.*

If the support of a fuzzy set is restricted and countable, the notation can be simplified as $\tilde{A} = \bigcup_{x \in U} {}^{\mu_{\tilde{A}}}\!/_{x}$.

Example 7. If we want the political party to be a fuzzy set, we have to assign the membership function for each legislator, depending on the expected value he/she will contribute to party voting. If, for example, the political party $\tilde{A}$ is marked as $\tilde{A} =$

$\left\{ {}^{0.7}/_{\text{Adam}}, {}^{1}/_{\text{David}}, {}^{1}/_{\text{Jane}}, {}^{0.2}/_{\text{Betty}}, {}^{0.1}/_{\text{Quinn}} \right\}$, it consists of 5 members who are creating the support of fuzzy set: $Supp(\tilde{A}) = \{\text{Adam, David, Jane, Betty, Quinn}\}$. The core of this fuzzy set consists of two members $Core(\tilde{A}) = \{\text{David, Jane}\}$.

In the parliamentary voting, the core will cover all legislators, who are never defecting in their voting in party loyalty or presence. The uncertainty in party loyalty and the members' presence of the respective political party as a whole can be expressed by a cardinality or relative cardinality of the fuzzy set.

Definition 4. *For a finite fuzzy set $\tilde{A} = (U, \mu_{\tilde{A}})$, the cardinality $|\tilde{A}|$ is defined as $|\tilde{A}| = \sum \mu_{\tilde{A}}$. Moreover, $\|\tilde{A}\| = \frac{|\tilde{A}|}{|x|}$ is called the relative cardinality of $\tilde{A}$.*

In the computation of voting power indices, it is easier to use crisp numbers. Therefore, the uncertainty caused by defecting of legislators in loyalty and party presence can be introduce using probabilistic or possibilistic method to the relative cardinality of fuzzy set. The uncertainty caused by creation of hidden coalitions can be incorporated using fuzzy set of all possible variations for the Shapley-Shubik power index and the fuzzy set of all coalitions for the Banzhaf power index.

6. Uncertainty in power indices computations: estimations and results

The recalculation of power indices with respect to three sources of uncertainty was done using the concept of a-priori coalitional structure, as this concept partially described the situation in the 1996–1998 Lower House of the Czech Parliament. Calculations were done for two parliamentary periods, 1998–2002 and 2002–2004. The uncertainty issues were taken from the preceding periods 1996–1998 and 1998–2002, respectively.

Parliamentary Period 1998–2002

For computation of power indices for the 1998–2002 period, we used the data of preceding, 1996–1998 period. The relative cardinality for each political party is derived as the multiplication of loyalty index and presence index. The used cardinalities are given in Table 17.

Table 17. Cardinalities used for computation of power indices for 1998–2002 period

Party	ODS	CSSD	KDU-CSL	KSCM	US
Members	63	74	20	24	19
Relative cardinality	0.858	0.810	0.862	0.849	0.825
Cardinality	54.05	59.95	17.24	20.38	15.68

For hidden coalitions, the measure of voting similarities was used to evaluate all members of all variations for computations of Shapley-Shubik index, and all possible coalitions for computations of Banzhaf power index. Recalculated power indices together with original power indices are given in Table 18.

Table 18. Old and recalculated power indices for 1998–2002

Power Index	ODS	CSSD	KDU-CSL	KSCM	US
Shapley-Shubik	0.300	0.300	0.133	0.133	0.133
Banzhaf	0.286	0.286	0.143	0.143	0.143
Adjusted Shapley-Shubik	0.379	0.396	0.067	0.089	0.067
Adjusted Banzhaf	0.390	0.409	0.067	0.067	0.067

The correlation coefficient of recalculated power indices with coefficient of success stayed unchanged; values are 0.352 and 0.394 for Shapley-Shubik and Banzhaf power index, respectively. However, values of rank correlation coefficient increased up to 0.45 for the Banzhaf power index.

Parliamentary Period 2002–2004

The 2002–2004-power indices' computation is based on the data from 2002–2004 period. The relative cardinality for each political party is derived as the multiplication of loyalty index and presence index. The used cardinalities are given in Table 19.

Table 19. Cardinalities used for computation of power indices for 2002–2004 period

Party	ODS	CSSD	KDU-CSL	KSCM	US
Members	58	70	23	41	8
Relative cardinality	0.766	0.781	0.729	0.790	0.667
Cardinality	44.45	54.64	16.77	32.39	5.34

Expression of hidden coalitions is based on the measure of voting similarities. The voting similarities were used to evaluate each member's position in permutation for the Shapley-Shubik index, and to evaluate all possible coalitions for Banzhaf power index. For the discussed parliamentary period, the calculations improved power indices with a-priori coalitional structure. Old and recalculated power indices for 2002–2004 parliamentary period are given in Table 20.

Table 20. Old and recalculated power indices for 2002–2004

Power Index	ODS	CSSD	KDU-CSL	KSCM	US
Shapley-Shubik	0.233	0.400	0.067	0.233	0.067
Banzhaf	0.231	0.385	0.077	0.231	0.077
Recalculated Shapley-Shubik	0.191	0.452	0.145	0.167	0.046
Recalculated Banzhaf	0.229	0.383	0.135	0.199	0.053

Correlation coefficients of recalculated power indices with success coefficient increased as demonstrated in the table of correlations. Both original and recalculated power indices comparison is given in Table 21.

Table 21. Correlation coefficients of party success with power indices

	Pearson's corr. coeff. Index of Success	Rank corr. coeff. Index of Success
Original Shapley-Shubik	0.495	−0.527
Original Banzhaf index	0.495	−0.527
Recalculated Shapley-Shubik	0.716	0.700
Recalculated Banzhaf index	0.773	0.700

The recalculations of power indices are based on uncertainty emerged from defection of legislators in party loyalty, presence, and the uncertainty caused by creation of hidden coalitions. This uncertainty analysis exhibited positive shift in description of power distribution estimation, as demonstrated by correlation coefficient increase. However, these correlation coefficients are still not statistically significant. Therefore, there are more factors that have to be incorporated into the model. The improvement of this approach can lead to better predictability of future power distribution in legislative process.

7. Conclusion

The main aim of this article was to study the power of legislators in the Lower House of the Czech Parliament in 1996–2004 with respect to power distribution and its uncertainty. Power distribution is usually expressed by power indices. However, by comparing the a-priori power indices and the future voting success, the discrepancy in correlations of these two variables appeared. These results led to the idea, that there are more factors that can be incorporated into concept of power indices for better expression of future power distribution. This paper studied three possible sources of uncertainty—defection of legislators in their presence, party loyalty and creation of hidden coalition. The recalculations of power indices with respect to these sources of uncertainty displayed the positive shift in correlation of power indices and success index, but this shift is still not sufficient.

The improvement of this approach could lead to better predictability of future power distribution in legislative process. The model can be improved by incorporation of seniority of legislators and personal uncertainty of reelected members. Another approach to improvement of the power distribution estimation could be based on fuzzy relations on fuzzy numbers of legislator's weights.

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