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Tail Behavior of the Central European Stock Markets during the Financial Crisis

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Abstract In the paper we research statistical properties of the Central European stock markets. We focus mainly on the tail behavior of the Czech, Polish, and Hungarian stock markets and compare them to the benchmark U.S. and German stock markets. We fit the data of the 4-year period from March 2005 to March 2009 with the stable probability distribution model and discuss its tail behavior. As the estimation of the tail exponent is very sensitive to the size of the data set, the estimates can be misleading for short daily samples. Thus, we employ high-frequency 1-minute data, which proves to be a good choice as it reveals interesting findings about the distributional properties. Furthermore, we study the difference in stock market behavior before and during the financial crisis.

Keywords Financial crisis, tail behavior, stock markets, stable probability distribution
JEL classification G14, C13, C16

1. Introduction

Statistical analysis of financial data has been of vigorous interest in recent years, mainly among the physics community (Mantegna and Stanley 2000; Bouchaud and Potters 2001; Buchanan 2002; Mantegna et al. 1999; Plerou et al. 2000; Stanley et al. 2000; Stanley 2003). One of the main reasons driving the research is the use of established statistical characteristics to better describe and understand the real-world financial data.

Dynamics of financial markets is the outcome of large number of individual decisions based on heterogeneous information. Financial returns representing the interaction of the market participants have been assumed to be normally distributed for a long time. The strongest argument supporting this assumption is based on the Central Limit Theorem, which states that the sum of a large number of independent, identically distributed variables from a finite-variance distribution will tend to be normally distributed. However, financial returns showed to have heavier tails, which is a possible source of infinite variance.

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Mandelbrot (1963) and Fama (1965) proposed stable distributions as an alternative to the Gaussian distribution model. Stable distributions were introduced by Lévy (1925), who investigated the behavior of sums of independent random variables. Although we know other heavy-tailed alternative distributions (such as student's t, hyperbolic or normal inverse Gaussian), stable distributions are attractive for researchers as they are supported by the generalized Central Limit Theorem. The theorem states that stable laws are the only possible limit distributions for properly normalized and centered sums of independent, identically distributed random variables. A sum of two independent random variables having a Lévy-stable distribution with parameter α is again a Lévy-stable distribution with the same parameter α. However, this invariance property does not hold for different values of α. Observed stock market prices are argued to be the sum of many small terms, hence a stable model should be used to describe them. When $\alpha < 2$, the variance of the stable distribution is infinite and the tails are asymptotically equivalent to a Pareto law, i.e., they exhibit power-law behavior. Stable distributions have been proposed as a model for many types of physical and economic systems as they can accommodate fat tails and asymmetry and fit the data well. Examples in finance and economics are given in Mandelbrot (1963), Fama (1965), Embrechts et al. (1997) or Rachev and Mittnik (2000).

There have, however, been many applications of Lévy-stable distributions to empirical data sets which could raise doubts about the correctness of the tail estimate (Lux 1996; Voit 2005). There is a significant difference between the value of the estimated α (based on the whole data set by estimation of stable distribution parameters) and the estimated tail exponent, introduced in extreme-value theory. The tail exponent is estimated only on an arbitrarily chosen part of the data, i.e. by the Hill estimator (Hill 1975; Weron 2001). Since extreme observations of prices on financial markets are of great importance, this problem deserves further research. If α is underestimated, the occurrence of extreme events is overestimated. Weron (2001) shows that the estimated tail exponent is very sensitive to changes in parameters and to the size of the data set, hence the estimates can be highly misleading. Simulations show that a large data set (10^6) is needed for identification of the true tail behavior. The logical step is to use high-resolution data analysis. Lux (1996) was one of the first to use high-frequency data, doing so for analysis of the German stock market index. Several studies concerning estimation of stable distributions followed (Mantegna and Stanley 2000; Dacorogna et al. 2001; Voit 2005).

In our paper, we append an analysis of Central European stock markets to the discussions. In the first part, we briefly discuss the basics of stable distributions and their tail behavior. In the second part, we follow with an empirical analysis of the daily returns of the Czech PX, Polish WIG, Hungarian BUX, German DAX, and U.S. SP500 stock market indices. We fit the stable distribution to the empirical distributions of all the indices during the period of March 2005 to March 2009. Moreover, we divide this period into two equal halves so we can compare the behavior before and during the crisis. After a discussion of the appropriateness of the stable model for our data and a description of the tail behavior, we employ high-frequency data in the analysis. We estimate the parameters of stable distributions for each day and use the intraday scale parameter estimate for standardization of the daily data. Finally, the high-frequency data allow us to study the standardized daily returns adjusted for volatility.

2. Stable distributions

Stable distributions are a class of probability laws with appealing theoretical properties. Their application to financial modeling comes from the fact that they generalize the Gaussian distribution, which does not describe well-known stylized facts about stock market data. Stable distributions allow for heavy tails and skewness. In this paper, we provide a basic idea about stable distributions. Interested readers can find the theorems and proofs in Nolan (2003), Zolotarev (1986), and Samorodnitsky and Taqqu (1994).

The reason for the term stable is that stable distributions retain their shape up to scale and shift under addition: if X, X_1, $X_2, \ldots, X_n$ are independent, identically distributed stable random variables, then for every n

$$X_1 + X_2 + \ldots + X_n \stackrel{d}{=} c_n X + d_n \tag{1}$$

for constants $c_n > 0$ and d_n. Equality ($\stackrel{d}{=}$) means that the right-hand and left-hand sides have the same distribution. Normal distributions satisfy this property: the sum of normals is normal. In general, the class of all laws satisfying (1) can be described by four parameters, $(\alpha, \beta, \gamma, \delta)$. Parameter α is called the *index* of the law or the *index of stability* or *characteristic exponent* and must be in the range $\alpha \in (0, 2]$. The coefficients c_n are equal to $n^{1/\alpha}$. Parameter β is called the *skewness* of the law and must be in the range $-1 \leq \beta \leq 1$. If $\beta = 0$, the distribution is symmetric, if $\beta > 0$ it is skewed to the right, and if $\beta < 0$ it is skewed to the left. While parameters α and β determine the shape of the distribution, γ and δ are scale and location parameters, respectively.

Due to a lack of closed form formulas for probability density functions (except for three stable distributions: Gaussian, Cauchy, and Lévy) the α-stable distribution can be described by a characteristic function which is the inverse Fourier transform of the probability density function, i.e., $\phi(u) = E\exp(iuX)$.

A confusing issue with stable parameters is that there are multiple parametrizations used in the literature. Nolan (2003) provides a good guide to all the definitions. In this paper, we will use Nolan's parametrization, which is jointly continuous in all four parameters. A random variable X is distributed by $S(\alpha, \beta, \gamma, \delta)$ if it has the following characteristic function:

$$\phi(u) = \begin{cases} \exp(-\gamma^{\alpha}|u|^{\alpha}[1 + i\beta(\tan\frac{\pi\alpha}{2})(\mathrm{sign}u)(|\gamma u|^{1-\alpha} - 1)] + i\delta u) & \alpha \neq 1 \\ \exp(-\gamma|u|[1 + i\beta\frac{2}{\pi}(\mathrm{sign}u)\ln(\gamma|u|)] + i\delta u) & \alpha = 1 \end{cases} \tag{2}$$

There are only three cases where a closed-form expression for density exists and we can verify directly if the distribution is stable—the Gaussian, Cauchy, and Lévy distributions. Gaussian law is stable with $\alpha = 2$ and $\beta = 0$. More precisely, $N(0, \sigma^2) = S(2, 0, \sigma/\sqrt{2}, 0)$. Cauchy law is stable with $\alpha = 1$ and $\beta = 0$, $\mathrm{Cauchy}(\gamma, \delta) = S(1, 0, \gamma, \delta)$; and finally, Lévy law is stable with $\alpha = 1/2$ and $\beta = 1$; $\mathrm{Lévy}(\gamma, \delta) = S(1/2, 1, \gamma, \gamma + \delta)$. Nolan (2003) shows these examples in detail.

For all values of parameter $\alpha < 2$ and $-1 < \beta < 1$, stable distributions have two tails that are asymptotically power laws. The asymptotic tail behavior of non-Gaussian stable laws for $X \sim S(\alpha, \beta, \gamma, \delta)$ with $\alpha < 2$ and $-1 < \beta < 1$ is defined as follows:

$$\lim_{x \to \infty} x^{\alpha} P(X > x) = c_{\alpha} (1 + \beta) \gamma^{\alpha} \tag{3}$$
$$\lim_{x \to \infty} x^{\alpha} P(X < -x) = c_{\alpha} (1 - \beta) \gamma^{\alpha},$$

where

$$c_{\alpha} = \sin\left(\frac{\pi\alpha}{2}\right) \Gamma(\alpha) / \pi. \tag{4}$$

A negative aspect of non-Gaussian stable distributions ($\alpha < 2$) is that not all moments exist.[1] The first moment EX is not finite (or is undefined) when $\alpha < 1$. On the other hand, when $1 < \alpha \leq 2$ the first moment is defined as

$$EX = \mu = \delta - \beta\gamma \tan\frac{\pi\alpha}{2}. \tag{5}$$

Non-Gaussian stable distributions do not have finite second moment. It is also important to emphasize that the skewness parameter β is different from the classical skewness parameter used for the Gaussian distribution. It cannot be defined because the second and third moments do not exist for non-Gaussian stable distributions. The kurtosis is also undefined, because the fourth moment does not exist either.

The index of stability α gives important information about financial market behavior. When $\alpha < 2$, extreme events are more probable than for the Gaussian distribution. From an economic point of view some values of parameter α do not make sense. For example, in the interval $0 < \alpha < 1$ the random variable X does not have a finite mean. In this case, an asset with returns which follow a stable law with $0 < \alpha < 1$ would have an infinite expected return. Thus, we are looking for $1 < \alpha < 2$ to be able to predict extreme values more precisely than by the Gaussian distribution.

There are several methods for estimating the parameters of stable distributions. For a detailed discussion, see Nolan (2003), McCulloch (1996). In our paper, we use maximum likelihood estimation of the parameters.

3. Daily data

In this section, we will use real-world data sets consisting of Central European, U.S. and German stock market indices in order to study their distributions. More precisely, we use 1,000 daily prices from March 2005 until March 2009 of value-weighted PX, BUX, WIG, DAX, and SP500 indices, representing an approximation of the Czech, Hungarian, Polish, German, and U.S. stock market indices, respectively.

Figure 1 shows the plots of all the indices. In our analysis, we examine the differences in the distributions of two groups: the Central European stock markets as represented by PX, BUX, and WIG and DAX and SP500 as benchmarks (plotted in

[1] It is possible to define a fractional absolute moment of order p, where p is any real number. For $0 < p < \alpha$, $E|X|^p$ is finite, but for $p \geq \alpha$, $E|X|^p = \infty$ (Nolan 2003).

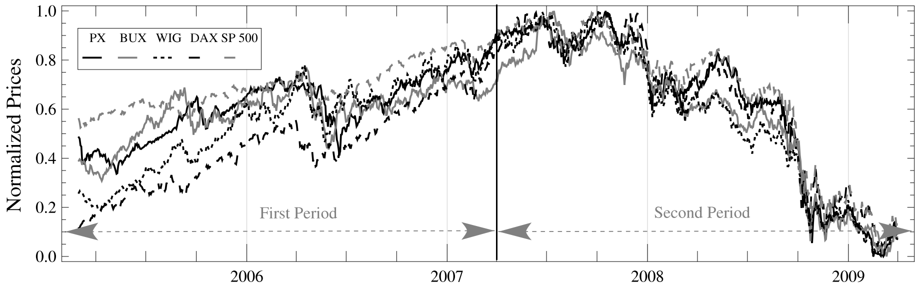


Figure 1. Prices of the indices examined, normalized to the $[0,1]$

Figure 1). We also divide the data set into two subsets, the first one before the first quarter of 2007 and the other one during the crisis. We will refer to these two groups as the *first period* and the *second period* in the text. The division is logical as it allows us to study the differences in behavior of all the stock market indices before the crisis during the steady state growth period, and during the crisis. We can also compare the differences across the two groups of stock markets studied.

4. Empirical probability densities

The logarithmic daily changes of all the indices are fitted with stable probability densities. For the estimation, we use John P. Nolan's STABLE[2] program, which can compute stable densities, cumulative distribution functions, and quantiles. We use the maximum likelihood method, which yields reliable results for our sample size (Nolan and Surami 1999). The fitted stable parameters of $S(\alpha,\beta,\gamma,\delta)$ in (2) are listed in Table 1. The fits are divided according to the periods examined. The fit for the whole period is given first, followed by the fits for the *first period* and *second period* subsamples.

Looking at whole period, we can observe that for all markets parameter α is less than 2, indicating that the fitted stable density might be non-Gaussian. All distributions show asymmetry to some extent. The scale parameter γ is significantly different from zero for all fits and the location parameter δ is close to zero for all markets.
Figure 2 completes the information about the fits, as it provides the stable density fits with histograms for all the stock market indices and periods tested.

For the whole tested period of 2005 to 2009, WIG and BUX have the highest estimated α. PX does not seem to belong in this group, as it shows a lower α which is not significantly different from the benchmark DAX index. On the other hand, the other benchmark index, SP500, has a significantly lower α than all the other indices. This is also valid when we divide the period into two sub-periods. Thus, we can see that the most developed and the most liquid U.S. stock market, as represented by SP500 index, departs from normality ($\alpha = 2$) and also from the group of Central European

[2] The STABLE program is available at http://academic2.american.edu/ jpnolan/stable/stable.html.

stock markets. Interestingly, DAX shows very similar estimates of stable parameters compared to the Central European indices.

Table 1. Parameter estimates of stable distribution for the individual indices

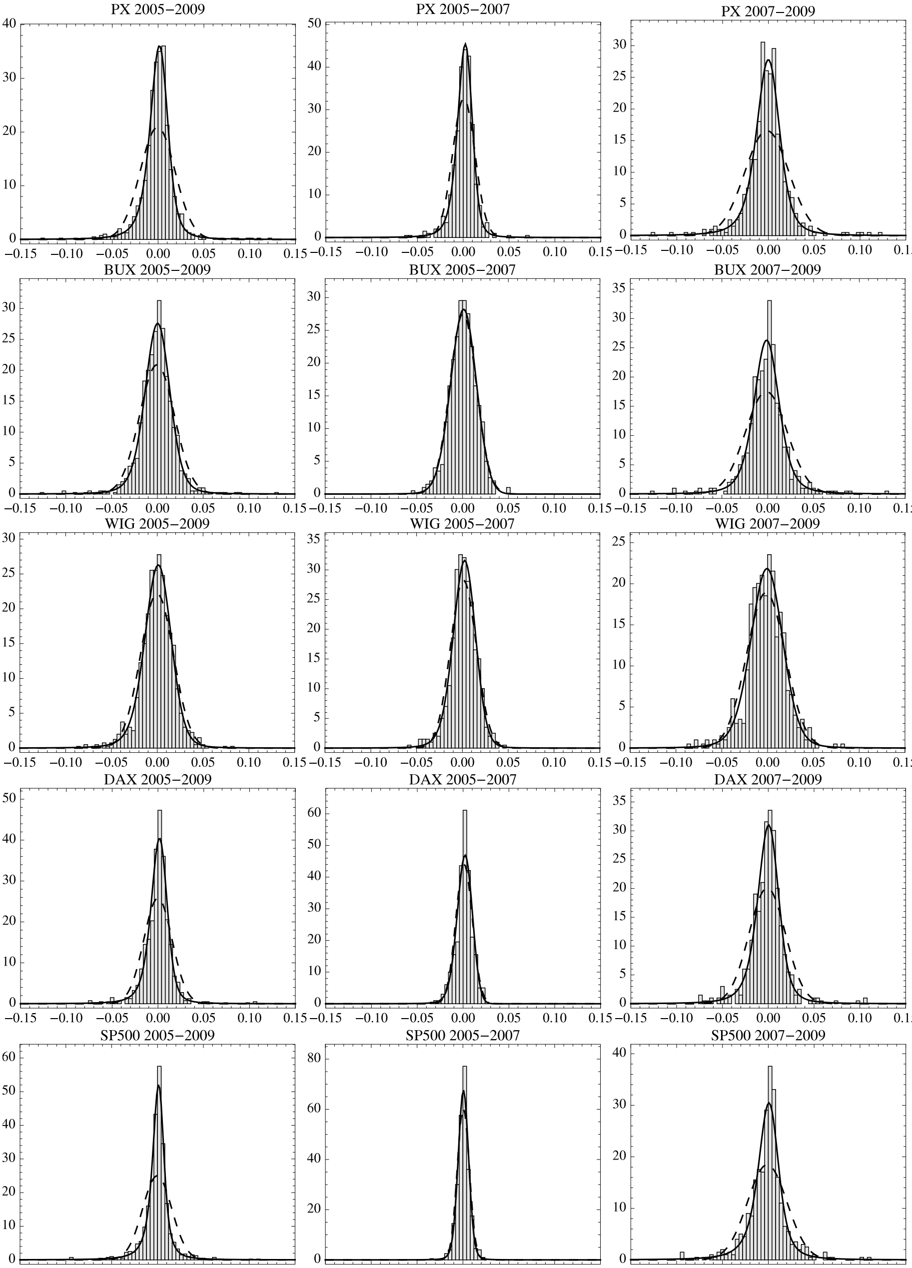
	α	β	γ	δ
Whole Period (2005–2009)				
PX	1.4790 (0.0974)	−0.2540 (0.1739)	0.0079 (0.0005)	0.0010 (0.0008)
BUX	1.6986 (0.0941)	−0.1393 (0.2610)	0.0103 (0.0006)	0.0001 (0.0011)
WIG	1.7424 (0.0900)	−0.3001 (0.2885)	0.0108 (0.0006)	0.0009 (0.0011)
DAX	1.5309 (0.0973)	−0.3093 (0.1857)	0.0071 (0.0005)	0.0013 (0.0008)
SP 500	1.2873 (0.0933)	−0.2021 (0.1387)	0.0056 (0.0004)	0.0008 (0.0006)
First Period (2005–2007)				
PX	1.6055 (0.1363)	−0.3848 (0.2913)	0.0063 (0.0006)	0.0019 (0.0009)
BUX	1.9098 (0.0912)	−0.5716 (0.7837)	0.0098 (0.0007)	0.0011 (0.0014)
WIG	1.8543 (0.1048)	−0.5964 (0.5619)	0.0090 (0.0007)	0.0022 (0.0014)
DAX	1.8712 (0.0891)	−0.9447 (0.2367)	0.0059 (0.0004)	0.0020 (0.0009)
SP 500	1.8515 (0.1100)	−0.1233 (0.6411)	0.0042 (0.0003)	0.0005 (0.0006)
Second Period (2007–2009)				
PX	1.4974 (0.1387)	−0.1607 (0.2575)	0.0103 (0.0009)	−0.0004 (0.0016)
BUX	1.5597 (0.1390)	−0.0403 (0.2922)	0.0109 (0.0009)	−0.0011 (0.0016)
WIG	1.7945 (0.1218)	−0.2262 (0.4746)	0.0130 (0.0010)	−0.0008 (0.0019)
DAX	1.5189 (0.1382)	−0.2339 (0.2622)	0.0092 (0.0008)	0.0001 (0.0014)
SP 500	1.3855 (0.1360)	−0.2007 (0.2202)	0.0095 (0.0009)	0.0002 (0.0014)

Note: The number in parenthesis is the 95% confidence interval half-width estimate.

The division of the data sets into the *first period* and the *second period* shows us more interesting results. All the α estimates of the *second period* are lower when compared to the *first period*, while the α of BUX, DAX, and SP500 are significantly lower. This indicates that from the Central European group, only the BUX index has significantly different tail behavior during the crisis than before the crisis.[3] On the other hand, the more developed stock markets in Germany and the USA show significantly heavier tails during the *second period*. This observation is confirmed by Figure 2. From Figure 2, we can also observe that the distributions from the *first period* are better described by the normal distribution, while the distributions from the *second period* of the financial crisis clearly depart from the normal distribution.

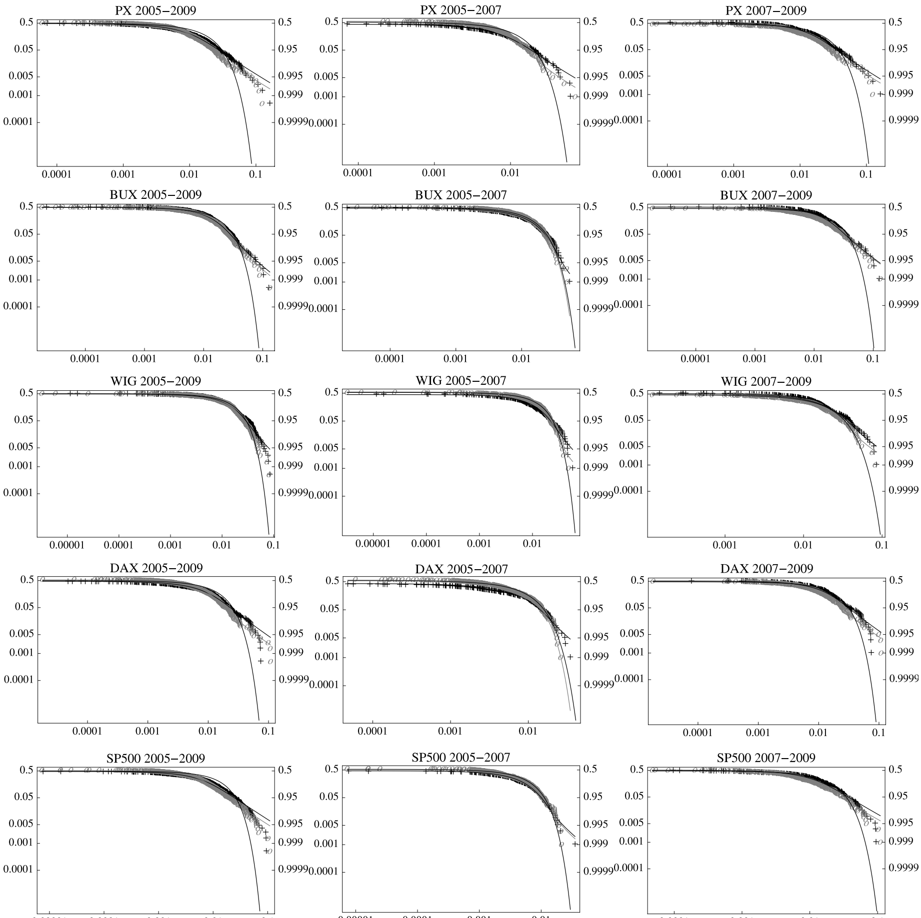
The Jarque-Bera statistics also strongly reject normality of all the stock market indices for the whole period. When we divide the period into two sub-samples, for the first period the Jarque-Bera statistics show departure from the normal distribution at the 99% significance level for all indices except BUX. For BUX, they reject the null of normality at 95%. For the *second period*, the Jarque-Bera statistics again strongly reject normality for all the stock market indices examined.

[3] This result is most likely influenced by the severe economic problems that started before the current financial crisis.



Note: Stable probability density function in solid lines, normal distribution function in dashed lines.

Figure 2. Stable probability density functions, normal distribution function and histograms of PX, BUX, WIG, DAX, and SP500



Note: The absolute value of the left tail is illustrated as black "+" and the absolute value of the right tail as grey "*o*". For comparison, the tail of the normal distribution is in black and is not linear.

Figure 3. Log-log plots of stable distribution fits for all indices for different periods

5. Tail behavior

While the fit of the stable parameters to the data set seems reasonable, it is not surprising that one can fit a data set with a 4-parameter stable model better than with a 2-parameter normal model. Thus, a relevant question is whether or not the stable fit describes the data better. For this purpose, goodness-of-fit diagnostics are used. The first one is a simple density plot and the second one is a variance-stabilized PP plot.[4] Figure 2 shows us the density plots with histograms. We can observe that the stable fit

[4] Probability-Probability plot (PP plot), plots two cumulative distribution functions against each other and thus allows for assessing how closely two data sets agree.

describes the distributions more reasonably than the normal distribution for the *second period*. We cannot really conclude this for the *first period* (these results are confirmed by the more rigorous analysis in the previous section). The PP plot of all fits is on the diagonal,[5] which also confirms the good fit of the model used.

The previous analysis did not show us a lot of details about the tails. The estimated α exponents may provide a clue to the tail behavior, but more rigorous analysis is needed to draw any conclusions. The tails are the most important part of the distribution we are interested in when studying financial data, so the motivation to take a closer look is really strong. First of all, we will use log-log fits to the cumulative distribution function. Stable distributions with $\alpha < 2$ should show linear parallel tails with a slope of minus α. Moreover, if β is zero, the tails are superimposed. When $\beta = \pm 1$, the lighter tail is not linear. Figure 3 shows the log-log plot[6] for our tested grid of stock market indices and periods. We also provide the tail of the normal distribution calculated from the sample, which is not linear.

From Figure 3 one can observe that the tails of the actual data are slightly lighter than the tail exponents from the stable fits. There are points visible on the right side of the graphs that lie below the two straight lines that characterize the stable fit. These points represent the most extreme observations on both the left and right tails of the distribution. An imaginary line drawn through these few points on the tails has a steeper slope than the line representing the stable fit (a higher absolute value of α). To provide a deeper and more rigorous insight into the tail behavior, we need more sophisticated methods of tail exponent estimation.

There are basically two tail index estimation methods. The first is based on log-log linear regression of the cumulative distribution function and the second is the Hill estimator (Hill 1975). The Hill estimator tends to overestimate the tail exponent of a stable distribution if α is close to two and the sample size is not very large. In general, both methods are very sensitive to the choice of parameters. For a more detailed discussion see Barunik and Vacha (2010), Weron (2001) and Embrechts et al. (1997). In our analysis we use the first method, calculating the tail exponent estimates from the upper and the lower 2% data tails.[7] The results are provided in Table 2. All the values of the tail exponent estimates are higher than the stable parameter estimates α from the previous section listed in Table 1.

Generally, we can see that the tail exponents from the *second period* are lower than the tail exponents from the *first period*, which confirms our previous result that the stock market distribution from the pre-crisis period is closer to normal distribution and the *second period* can be better described by stable distributions, which catch the heavy tails. But none of the estimates of the tail exponent α is less than 2. According to the generalized central limit theorem, sums of random variables from distributions with linear tail exponents that are less than two will converge to a stable distribution. Those with higher tail exponents will converge to a normal distribution. Thus, none of our data comes from a stable distribution. Moreover, if we take a simulation derived from

[5] We do not provide all the PP plots here for reasons of space. They can be provided on request.

[6] The log-log plot is a two-dimensional graph that uses logarithmic scales on both the horizontal and vertical axes. The plot shows a log-log fit to the cumulative distribution function.

[7] Amount has been chosen arbitrarily corresponding to the sample size.

Table 2. The tail exponents α for all the stock indices and periods

	Whole Period		First Period		Second Period	
	Left	Right	Left	Right	Left	Right
PX	2.0484	2.8222	2.4851	3.7801	2.2089	2.7226
BUX	2.8735	2.5276	4.5662	6.0168	2.9692	3.6715
WIG	4.3111	4.2127	7.3268	6.3841	4.0577	5.7020
DAX	2.0199	4.7485	14.048	4.9252	2.4838	5.5519
SP500	2.9095	3.4848	8.2624	3.3343	3.1789	3.7717

the fit of our data (Table 1) and reconstruct a price series, we find extreme changes more frequently than are ever found in a real financial time series. Thus, stable fit seems to overestimate the frequency of extreme events for financial data.

While the value of the tail exponent α has its asymptotic limit 2, it turns out to provide quite restrictive support for conclusions about the tail behavior (Weron 2001). Using simulations, Weron shows that reported values of the tail exponent α of around 3 may very well indicate a Lévy-stable distribution with $\alpha \approx 1.8$. Log-log linear regression is also very sensitive to the sample size and the choice of the number of observations used in the regression. Moreover, a slope of around -3 (tail exponent α equal to 3) may indicate non-Lévy-stable power-law decay in the tails or, on the contrary, a Lévy-stable distribution with $\alpha \approx 1.8$.

Weron (2001) also shows that for a typically sized data set of 10^4 or less, the plot may be quite misleading. The true tail behavior of stable laws is visible only for extremely large data sets, which leads us to the use of high-frequency asset returns. Moreover, we have to keep in mind that the choice of observations used in the regression is subjective and can yield large estimation errors.

6. High Frequency Data

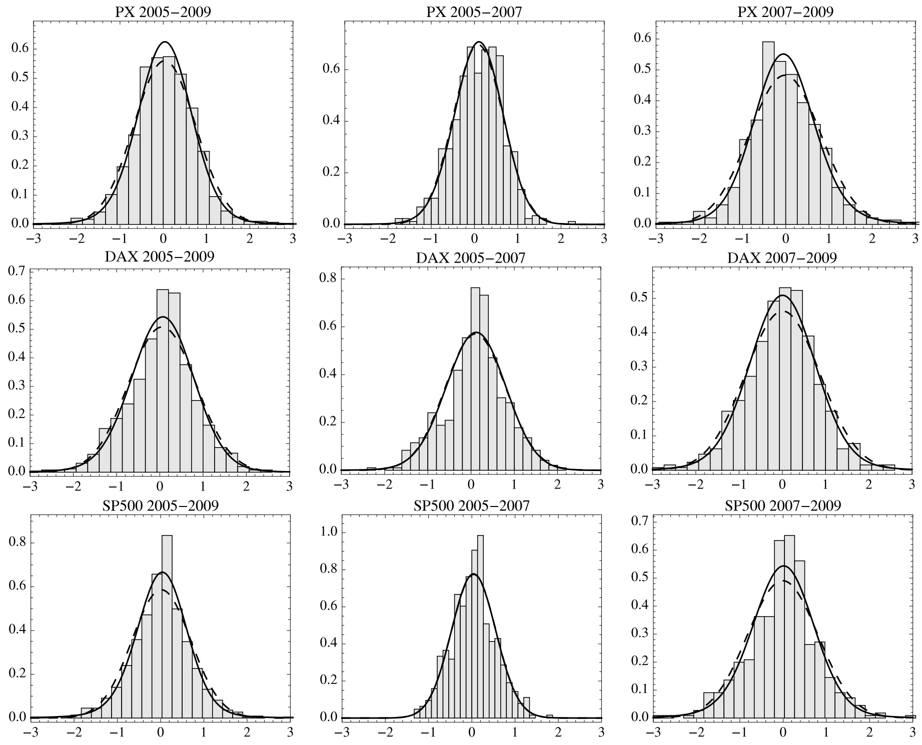
The previous analysis suggests that the stable model seems to be more appropriate for the tested data than the normal one, but we are not able to prove it as we are not able to draw conclusions about the tail behavior. We follow our experiment with the assumption that there is a structure that can be observed in the volatility of the financial time series. Sta-ble distributions have the mathematical property that the absolute mean deviation of a sample from the stable distribution $S(\alpha, \beta, \gamma, \delta)$ is proportional to the scale factor, γ.

To explore the data-generating process in further detail, we employ higher resolution data.[8] More precisely, we use 1-minute prices. We only have access to PX, DAX, and SP500 1-minute prices for the period of March 2005 until March 2009, so we decided it would be sufficient to continue the analysis leaving out the BUX and WIG

[8] There are several studies concerning the estimation of stable distributions on high frequency data. For example, tail exponents of the German stock market are examined in Lux (1996). Detailed treatments of the topic can be found in Mantegna and Stanley (2000), Dacorogna et al. (2001), and Voit (2005).

indices. The trading days are concatenated so that there is no gap between a closing price and the next open.

For each day we compute the parameters of the stable distribution so we obtain 1,000 daily estimates of all the parameters. The variation of the intraday scale factor γ can be viewed as an estimate of the daily volatility of the series. Thus, we scale the daily returns by the square root of the γ estimate. Figure 4 shows the histograms of these standardized data as well as the stable and normal fits. Table 3 shows the parameter estimates of the stable fits to these scaled daily series.



Note: Stable probability density function in solid lines, normal distribution function in dashed lines.

Figure 4. Stable probability density functions, normal distribution function and histograms of PX, BUX, WIG, DAX, and SP500

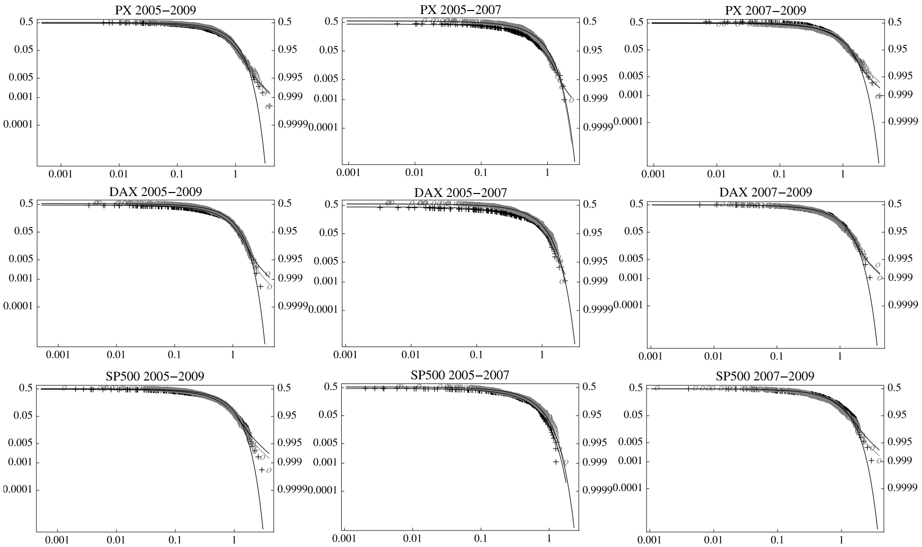
We can observe immediately that all the α estimates are larger than the α estimates of the unstandardized data (Table 1). Again, we are interested in the details of the tails. Figure 5 shows the left and right tails of all the estimates of the standardized data. All of them are closer to the stable fit when compared with the fits of the raw data in Figure 3. Jarque-Berra statistics strongly reject at the 99% significance level normality of all the data for the whole sample. The statistics are much lower than the Jarque-Berra values for the unstandardized daily data. Interestingly, the test does not reject normality of the data at the 95% level for the PX series and at 90% for the DAX and

SP500 series during the *first period*. For DAX and SP500, the statistics are very close to 1. For the *second period*, the Jarque-Berra statistics again strongly reject at the 99% significance level the null of normality of all the tested data.

Table 3. Stable parameter estimates for standardized indices by estimates of γ

	α	β	γ	δ
		Whole Period (2005–2009)		
PX	1.8610 (0.0740)	−0.0678 (0.4617)	0.4518 (0.0233)	0.0369 (0.0369)
DAX	1.8813 (0.0738)	−0.3046 (0.5138)	0.51581 (0.0275)	0.0724 (0.0553)
SP 500	1.8232 (0.0843)	−0.2049 (0.4002)	0.4276 (0.0241)	0.0343 (0.0471)
		First Period (2005–2007)		
PX	1.9207 (0.0818)	−0.6774 (0.7652)	0.3908 (0.0265)	0.1085 (0.0548)
DAX	2.0000 (0.0000)	−0.4288 (0.2367)	0.49401 (0.0313)	0.1020 (0.0626)
SP 500	1.8515 (0.1100)	−0.1233 (0.6411)	−0.0878 (0.0231)	0.4320 (0.0463)
		Second Period (2007–2009)		
PX	1.8486 (0.1063)	0.2580 (0.6005)	0.5169 (0.0379)	−0.451 (−0.0751)
DAX	2.0000 (0.0000)	−0.0878 (0.7064)	0.5535 (0.0424)	0.0061 (0.0843)
SP 500	1.8558 (0.1114)	−0.2250 (0.6622)	0.5233 (0.0406)	0.0603 (0.0806)

Note: The number in parenthesis is the 95% confidence interval half-width estimate.



Note: The absolute value of the left tail is illustrated as black "+" and the absolute value of the right tail as grey "*o*". For comparison, the tail of the normal distribution is in black and is not linear.

Figure 5. Log-log plots of stable distribution fits for all standardized indices for different periods

7. Conclusion

In this paper, we researched the statistical behavior of two groups of data; a Central European group, represented by the Czech PX, Polish WIG, and Hungarian BUX indices, and the benchmark German DAX and U.S. SP500. Moreover, we divided the tested period of March 2005 to March 2009 into two sub-periods. The first half of the data represents the pre-crisis period and the second half comprises data of the current world financial crisis. We fit the stable distribution to all the data sets and sub-periods.

The first period is better described by the normal distribution in comparison with the second period (except for the PX index). When we explore the tail behavior in detail, we find that the stable distribution fits embody fatter tails than can be observed in the data. We thus arrive at the same result as other studies, namely, that the real data show fatter-than-normal but lighter-than-stable tails. In other words, the stable fit overestimates extreme events in the stock markets. We came across another tail estimation problem during the analysis: a large estimation error when using small samples. This problem can be addressed using high frequency data. As we cannot conclude that the analyzed daily returns come from stable random data, we continue the analysis.

For the further analysis, we employ 1-minute index returns. We use γ estimates for each day to standardize the daily index returns. The stable fits and tails of these standardized returns are better described by a stable distribution and show that stock market returns may be generated from a mixture distribution of random variables with stable distributions. When the volatility is adjusted out, what remains is a stable random model.

It seems reasonable to say that the price formation process emerges from sums of returns and these are driven by a fat-tailed distribution in market order books. This process becomes extremely complicated and does not have the convenient features of simple mathematical laws that might be attributed to independent and identically distributed random variables.

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